1. State the three “sub-properties” that must be true about the function \( f \) at the number \( a \) if \( f \) is continuous at \( a \). (6 points)

   i. 

   ii. 

   iii. 

2. State each value of \( x \) where the function \( f \) shown in Figure 1 is discontinuous. For each discontinuity, state (by number) each of the properties that you stated in problem 1 that fail at that value of \( x \). Please note that you will not get full credit for this question if your answer to question 1 is not 100% correct. (9 points)
3. Use limit laws to formally establish the value of \( \lim_{x \to -\infty} \frac{3x^3 + 2x}{3x - 2x^3} \). (12 points)

To earn full credit your work must include all relevant limit law steps accompanied by a statement of the limit law number(s) used in each of those steps (as shown in class).
4. Use limit laws to formally establish the value of \( \lim_{x \to \frac{\pi}{4}} \frac{\sin(x) - \cos(x)}{\cos(2x)} \). (12 points)

To earn full credit your work must include all relevant limit law steps accompanied by a statement of the limit law number(s) used in each of those steps (as shown in class).

Some useful information …

\[
\cos(2x) = \cos^2(x) - \sin^2(x) \text{ and } \sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}
\]
5. The function $z$ shown in Figure 2 was generated by the formula $y = 2 + 4x - x^2$.
   
   a. Simplify the difference quotient for $z$. (12 points)

   b. Use the graph to find the slope of the secant line to $z$ between the points where $x = -1$ and $x = 2$.
   Check your simplified difference quotient for $z$ by using it to find the slope of the same secant line.
   **Make sure that you show work that clearly indicates that you completed both tasks stated in this problem.** (6 points)
6. Write into each provided blank the number or symbol that makes the equation “true.” The correct symbol for some of the blanks may be \( \infty \) or \(-\infty\); all other correct answers are numbers. (2 points each)

   a. \( \lim_\infty \frac{e^{2x}}{e^{x^2}} = 
   \)

   b. \( \lim_{x \to -\infty} \frac{e^{2x}}{e^x} = 
   \)

   c. \( \lim_{x \to 2^+} \frac{x^2 - 4}{x^2 - 4x + 4} = 
   \)

   d. \( \lim_{x \to \infty} \sin \left( \frac{\pi e^{3x}}{2e^x + 4e^{3x}} \right) = 
   \)

   e. \( \lim_{h \to 0} \frac{5h^2 + 3}{2 - 3h^2} = 
   \)

   f. \( \lim_{h \to -\infty} \frac{5h^2 + 3}{2 - 3h^2} = 
   \)

7. If a ball is thrown straight up into the air with an initial velocity of 40 ft/s, then the height (ft) of the ball \( t \) seconds after it is thrown is given by the function \( s(t) = 40t - 16t^2 \).

   What, including unit, is the average rate of change in \( s \) over the interval \([2, 3]\) and what does this rate tell you about the motion of the ball? (9 points)
8. Draw onto Figure 3 a function, \( f \), that satisfies each and every one of the following properties. Make sure that you draw and appropriately label all asymptotes. (14 points)

- The only discontinuities on \( f \) occur at \(-4 \) and \( 3 \)
- \( f \) has no \( x \)-intercepts
- \( f \) is continuous from the right at \(-4 \)
- \( \lim_{x \to -4^-} f(x) = 1 \) and \( \lim_{x \to -4^+} f(x) = -2 \)
- \( \lim_{x \to 3^-} f(x) = -\infty \)
- \( \lim_{x \to \infty} f(x) = -\infty \)
- \( f \) has a constant slope of \(-2\) over \((-\infty, -4)\)

9. Consider the function

\[
  f(x) = \begin{cases} 
    x + 2 & \text{if } x < 0 \\
    1 + e^x & \text{if } 0 \leq x \leq 1 \\
    \frac{x - 4}{(x - 4)(x - 6)} & \text{if } x > 1
  \end{cases}
\]

State the values of \( x \) where each of the following occur. If a stated property doesn't occur, make sure that you state that (as opposed to simply leaving the answer space blank). (2 points each)

a. At what values of \( x \) is \( f \) discontinuous?

b. At what values of \( x \) is \( f \) continuous from the left but not the right?

c. At what values of \( x \) is \( f \) continuous from the right but not the left?

d. At what values of \( x \) does \( f \) have removable discontinuities?