To earn full credit on any given problem your work must be presented in a manner consistent with that demonstrated and discussed during both lecture and lab. Remember to show your algebraic steps one step at a time.

1. State the three "sub-properties" that must be true about the function \( f \) at the number \( a \) if \( f \) is continuous at \( a \). (6 points)

   \( f(a) \) must be defined

   \( \lim_{{x \to a}} f(x) \) must exist

   \( \lim_{{x \to a}} f(x) \) and \( f(a) \) must be equal

2. State each value of \( t \) where the function \( f \), shown in Figure 1 is discontinuous. For each discontinuity, state (by number) each of the properties that you stated in problem 1 that fail at that value of \( t \). Please note that you will not get full credit for this question if your answer to question 1 is not 100% correct. (12 points)

<table>
<thead>
<tr>
<th>( x )-value</th>
<th>Failing properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>i, ii, iii</td>
</tr>
<tr>
<td>1</td>
<td>ii, iii</td>
</tr>
<tr>
<td>2</td>
<td>i, iii</td>
</tr>
<tr>
<td>3</td>
<td>iii</td>
</tr>
<tr>
<td>5</td>
<td>i, ii, iii</td>
</tr>
</tbody>
</table>

Figure 1
3. Use limit laws to formally establish the value of \( \lim_{t \to a} \sqrt[3]{\frac{3t^2 + 75}{75 + 12t}} \). (15 points)

To earn full credit your work must include all relevant limit law steps accompanied by a statement of the limit law number(s) used in each of those steps (as shown in class).

\[
\lim_{t \to a} \sqrt[3]{\frac{3t^2 + 75}{75 + 12t}} = \lim_{t \to a} \sqrt[3]{\frac{3t^2 + 75}{75 + 12t} \cdot \frac{1/t^2}{1/t^2}} = \lim_{t \to a} \sqrt[3]{\frac{3 + 75/t^2}{75/t^2 + 12}}
\]

LL A 6

\[
= \sqrt{\lim_{t \to a} \frac{3 + 75/t^2}{75/t^2 + 12}}
\]

LL AS

\[
= \sqrt{\lim_{t \to a} \frac{3 + 75/t^2 + 12}{75/t^2 + 12}}
\]

LL A 1

\[
= \sqrt{\frac{3 + 0}{0 + 12}}\quad LL R^2, LL R 3
\]

\[
= \frac{1}{2}
\]
4. Use limit laws to formally establish the value of $\lim_{x \to 0} \frac{\sin(2x) + \cos(2x) - 1}{\sin(x)}$. (15 points)

To earn full credit your work must include all relevant limit law steps accompanied by a statement of the limit law number(s) used in each of those steps (as shown in class).

Some really useful information ... $\cos(2x) = 1 - 2\sin^2(x)$ and $\sin(2x) = 2\sin(x)\cos(x)$

$$\lim_{x \to 0} \frac{\sin(2x) + \cos(2x) - 1}{\sin(x)} = \lim_{x \to 0} \frac{2\sin(x)\cos(x) + (1 - 2\sin^2(x)) - 1}{\sin(x)}$$

$$= \lim_{x \to 0} \frac{2\sin(x)\cos(x) - 2\sin^2(x)}{\sin(x)}$$

$$= \lim_{x \to 0} \frac{2\sin(x)[\cos(x) - \sin(x)]}{\sin(x)}$$

$$= \lim_{x \to 0} \left(2 \left[ \cos(x) - \sin(x) \right] \right) \text{ LCA 7}$$

$$= 2 \lim_{x \to 0} \left( \cos(x) - \sin(x) \right) \text{ LCA 3}$$

$$= 2 \left[ \lim_{x \to 0} \cos(x) - \lim_{x \to 0} \sin(x) \right] \text{ LCA 2}$$

$$= 2 \left[ \cos(0) - \sin(0) \right] \text{ LCA 6}$$

$$= 2 \left[ \cos(0) - 0 \right]$$

$$= 2$$
5. Consider the function \( f(x) = x^2 \). Find a formula for the slope of the secant line connecting the point \((x, f(x))\) to the point \((x + h, f(x + h))\); completely simplify the formula. Show all relevant work in a manner consistent with that discussed and illustrated in class and the lab manual. (12 points)

\[
M_{sec} = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x + h; h \neq 0
\]

6. Draw onto Figure 2 a function, \( f \), that satisfies each and every one of the following properties. Make sure that you draw and appropriately label all asymptotes. (16 points)

- The only discontinuities on \( f \) occur at -3, 0, and 2.
- The only \( x \)-intercepts on \( f \) occur at 2 and -1.
- \( \lim_{x \to -3^-} f(x) = -1 \)
- \( \lim_{x \to -3^+} f(x) = -1 \) and \( \lim_{x \to -\infty} f(x) = -1 \)
- \( \lim_{x \to 0} f(x) = -\infty \)
- \( f \) is continuous on \([-3,0)\)
- \( f(-5) = 5 \) and \( f(-4) = 4 \)
- \( f \) has constant slope on \((-\infty, -3)\).

![Figure 2: f]
7. Consider the function \( g(x) = 3 - 5x \). Use the definition of the first derivative to find the formula for \( g'(x) \). (12 points)

\[
g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{3 - 5(x+h) - (3 - 5x)}{h}
\]

\[
= \lim_{h \to 0} \frac{3 - 5x - 5h - 3 + 5x}{h}
\]

\[
= \lim_{h \to 0} \frac{-5h}{h}
\]

\[
= \lim_{h \to 0} (-5)
\]

\[
= -5
\]
8. For each given statement circle T if the statement is true and circle F if the statement is false. This question refers to the function $f$ shown in Figure 3. (6.5 points total)

- T or F  
  $f$ is continuous from the right at $-4$.

- T or F  
  $f$ is continuous from the right at $1$.

- T or F  
  The discontinuity on $f$ at $2$ is removable.

- T or F  
  The discontinuity on $f$ at $3$ is removable.

- T or F  
  The discontinuity on $f$ at $5$ is removable.

- T or F  
  $f$ is continuous on $(-\infty, -4]$.

- T or F  
  $f$ is continuous on $(1, 2)$.

- T or F  
  $f$ is continuous on $[3, 5]$.

- T or F  
  $\lim_{t \to 3} f(t)$ exists.

- T or F  
  $\lim_{t \to 5} f(t)$ exists.

- T or F  
  $\lim_{t \to 5} \frac{1}{f(t)}$ exists.

- T or F  
  $\lim_{t \to 5} \frac{f(t)}{f(t)}$ exists.

- T or F  
  $\lim_{t \to \infty} f(t)$ exists.

9. If $v(t) = 12t$ is the speed of an object (in ft/s) where $t$ is the amount of time that has passed since noon (measure in s), what, including unit, is the value of $\frac{v(5) - v(3)}{5s - 3s}$? (5.5 points)

$$\frac{v(5) - v(3)}{5s - 3s} = \frac{60 - 36}{2s} = 12 \cdot \frac{5}{s}$$