Drawing Functions based upon Derivative Sign Information

Table 1: 4 Basic Curve Shapes and the attendant Derivative Signs

<table>
<thead>
<tr>
<th></th>
<th>$f'(x) &gt; 0$</th>
<th>$f'(x) &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f''(x) &gt; 0$</td>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
<tr>
<td>$f''(x) &lt; 0$</td>
<td>[Diagram]</td>
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</tr>
</tbody>
</table>

Example
Sketch onto Figure 1 a continuous curve, $y = f(x)$, that has the following properties.

- $f(0) = 1$
- $f'(x) < 0$ over $(-\infty, -2)$ and $(3, \infty)$
- $f''(x) > 0$ over $(-2,1)$ and $(1,3)$
- $f''(x) < 0$ over $(-\infty, -2)$ and $(1,\infty)$
- $f''(x) > 0$ over $(-2,1)$
Four different functions are shown in figures 2 - 5. In each question on this page a property is stated that is true for only one of the functions shown in figures 2 - 5. For each property, state the figure number that shows the function with the stated property. Write below each question a brief explanation of how you made your determination.

a. The function whose first derivative is positive over the interval $(2, 4)$ is shown in Figure ________

b. The function whose first derivative is decreasing over the interval $(2, 4)$ is shown in Figure ________

c. The function whose antiderivative is increasing over $(2, 4)$ is shown in Figure ________

(Please note: $F$ is an antiderivative of $f$ if and only if $F'(x) = f(x)$.)

d. The function whose first derivative has a local maximum point at $x = 3$ is in Figure ________
e. The function whose antiderivative is concave down over \((2,4)\) is shown in Figure _____
(Remember: \(F\) is an antiderivative of \(f\) if and only if \(F' (x) = f (x)\).)

f. The function whose antiderivative has a local maximum point at \(x = 4\) is in Figure _____

The graph of a first derivative function, \(y = g'(x)\), is shown over \((-4, 4)\) in Figure 6. Sketch onto Figure 7 the continuous function \(y = g(x)\), over \([-4, 4]\) given that \(g(-4) = -4\) and \(g(0) = 4\).
Figure 8 is the graph of a function named $f$. $F$ is an antiderivative of $f$; that is, $F'(x) = f(x)$. Answer each of the following questions about $F$.

- On what intervals is $F$ decreasing? Explain.
- At what values of $x$ does $F$ have a local extreme value? Explain.
- At what values of $x$ does $F$ have an inflection point? Explain.

Suppose that $g'(4) = 11$ and $g''(t) = -2$ for all values of $t$. Over what interval(s) is the function $y = g(t)$ decreasing? Explain!
Water flows at a constant rate into a large conical tank (pointy end down 😄). Let $V(t)$ ($\text{ft}^3$) and $H(t)$ (ft) be, respectively, the volume of water in the tank and the height of the water in the tank $t$ minutes after the water begins to flow. Suppose that five minutes after the water begins to flow the tank is one quarter full. For each of the following expressions, state the units on the expression and state whether the value is positive, negative, or zero.

<table>
<thead>
<tr>
<th>Function value</th>
<th>Unit</th>
<th>Positive/Negative/Zero?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V'(5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V''(5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H'(5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H''(5)$</td>
<td></td>
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</table>

Figures A-F show 6 different functions. The first derivative of one of these functions is $f''(x) = 3 - |x|$. Which one is a graph of $y = f(x)$? No work need be shown.
Each of the following sentences is true if one of the words/phrases in Table 1 is inserted into the blank. Find the proper word/phrase for each of the blanks. Read each sentence carefully!!

<table>
<thead>
<tr>
<th>Table 1: Blank Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing</td>
</tr>
<tr>
<td>Concave Up</td>
</tr>
<tr>
<td>Positive</td>
</tr>
<tr>
<td>Zero</td>
</tr>
<tr>
<td>&quot;Positive, Zero or UD&quot; (This is one choice)</td>
</tr>
<tr>
<td>Horizontal</td>
</tr>
</tbody>
</table>

Note: UD is short for undefined

- If \( f' \) is negative at every value of \( x \) on \((2,5)\), then \( f' \) is __________________________ over the entire interval \((2,5)\).

- If \( f'' \) is positive at every value of \( x \) over \((2,5)\), then \( f'' \) is __________________________ over the entire interval \((2,5)\).

- If \( g' \) is increasing over the entire interval \((2,5)\), then \( g \) is __________________________ over the entire interval \((2,5)\).

- If the slope of \( g \) is increasing over the entire interval \((2,5)\), then \( g \) is __________________________ over the entire interval \((2,5)\).

- If the slope of \( g \) is positive over the entire interval \((2,5)\), then \( g \) is __________________________ over the entire interval \((2,5)\).

- If \( g \) is continuous and has a local minimum at the point where \( t = 11 \), then \( g' \) is __________________________ immediately to the right of \( t = 11 \).

- If the slope of \( g \) is increasing over the entire interval \((2,5)\), then at any point along the interval \((2,5)\), \( g'' \) is __________________________.

(This question is continued on page 7.)
If $f$ is differentiable and has a local maximum point at $x = 7$ then the tangent line to $y = f(x)$ at the point where $x = 7$ is ________________.

If $f$ is decreasing over the entire interval $(-4,4)$ then $f'(2)$ is ________________.

If $g'$ is concave up over the entire interval $(2,5)$, then $g''$ is ________________ over the entire interval $(2,5)$.

If the slope of $g'$ is negative over the entire interval $(-4,4)$, then $g''(2)$ is ________________.

If $g$ is concave up over the entire interval $(-4,4)$, then $g''(2)$ is ________________.

If $g$ has a local maximum at the point where $t = 1$, then the slope of the tangent line to $y = g(t)$ is ________________ at $t = 1$.

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Table 1: Blank Options

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Note: UD is short for undefined
The first derivative of a **continuous** function \( f \) is shown in Figure 9; that is, \( y = f'(x) \) is shown in Figure 9. Answer each of the following questions with regards to this function. The function \( F \) is some unknown antiderivative of \( f \).

**Note:** The correct answer to at least one of the questions is “there’s no way of knowing” or something to that effect.

a. Over what intervals is \( f \) linear? 
   b. Over what intervals is \( F \) linear?

   c. Over what intervals is \( f' \) increasing? 
   d. Over what intervals) is \( f'' \) increasing?

   e. Over what intervals is \( f \) negative? 
   f. Over what intervals is \( f'' \) negative?

   g. At what values of \( x \) is \( f \) nondifferentiable?

   h. At what values of \( x \) is \( F \) nondifferentiable?

   i. At what values of \( x \) is \( f'' \) undefined?
Svetlana was shopping at Trader Vladimers and put a carton of milk into her cart. Sadly, Svetlana did not notice that there was a slow leak in the bottom of the carton. Svetlana’s son Igor did notice the leak and decided to think about the calculus of the situation. Igor defined $V(t)$ to be the volume (qt) of milk remaining in the carton $t$ seconds after it was loaded into the cart and he pondered the following three questions. Please write down correct answers to each of the questions upon which Igor pondered.

a. What are the units on the output from the functions $V'$ and $V''$?

b. Only one of the following 3 values makes sense. What are the reasons the other two values don’t make sense.

$$V'(5) = -4 \text{ (unit)}, \ V'(5) = 4 \text{ (unit)}, \ V'(5) = -.04 \text{ (unit)}$$

c. What is the practical meaning (including units) of the meaningful derivative value from part (b) of this question?
The fuel consumption rate (measured in gallons per hour) of a certain car traveling at a speed of $v$ mph is given by the function $c = f(v)$.

a. What is the unit on $f''(40)$?

b. What is the unit on $f'''(40)$?

c. What is the practical meaning of $f'(40) = 2$ (unit)?

d. What is the practical meaning of $f''(40) = 0.03$ (unit)?