1. Use the definition of the first derivative to find the value of $f'(3)$ for the function $f(x) = 4x^2 - 2x$. Please note that you will get no credit for using a short cut formula to find the formula for $f'(x)$. In fact, I'm not looking for the first derivative formula, just the first derivative value at 3. (12 points)

$$f'(x) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{4(3+h)^2 - 2(3+h) - 30}{h}$$

$$= \lim_{h \to 0} \frac{4(9 + 6h + h^2) - 6 - 2h - 30}{h}$$

$$= \lim_{h \to 0} \frac{36 + 24h + 4h^2 - 6 - 2h - 30}{h}$$

$$= \lim_{h \to 0} \frac{22h + 4h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(22 + 4h)}{h}$$

$$= 22$$
2. Sketch onto Figure 2 a continuous antiderivative of the function \( f \) shown in Figure 1; specifically, draw the continuous antiderivative that passes through the point \((-4, 2)\). Please note that I am looking for a plausible curve, not the exact curve. (12 points)

3. Figure K shows a graph of Mr. Kitty's weight (lb) \( t \) weeks after the Cartmans brought him home from the SP no kill kitty shelter. Calculate, and state, the slope of the line—including unit— and interpret the slope as a rate of change. (7 points)

The slope is \( \frac{1}{3} \) lb/wk.

After coming home with the Cartmans, Mr. Kitty's weight increased at the constant rate of \( \frac{1}{3} \) lb/wk.
4. Use the definition of the first derivative to find the formula for \( g'(x) \) if \( g(x) = 2 \). Please note that you will get no credit for using a short cut formula to find the formula for \( g'(x) \). (8 points)

\[
g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{2 - 2}{h}
\]

\[
= \lim_{h \to 0} \frac{0}{h}
\]

\[
= 0
\]

5. Draw onto Figure 3 a plausible first derivative of a function, \( f \), that has each of the following properties. To be clear, I am telling you properties of \( f \) and you are going to draw \( f' \). (12 points)

- \( f \) is continuous over \((-7,7)\)
- \( f \) is increasing on \((-7,-2)\) and \((-2,4)\); \( f \) is decreasing on \((4,7)\).
- \( f \) is concave up on \((-4,-2)\), linear on \((-2,2)\), and concave down on both \((-7,-4)\) and \((2,7)\).
- The tangent lines to \( f \) at both \( x = -3 \) and \( x = -1 \) have a slope of 1.
- Over \((-7,7)\), \( f \) is nondifferentiable exactly one time.
- Over \((-7,7)\), there are exactly two points where the tangent line to \( f \) is horizontal.
6. Miss Communication makes a lot of embarrassing typos. The number of typos she makes in a given morning depends upon the amount of coffee she drinks (oz) before coming to work.

Figure T shows a graph of the number of embarrassing typos Miss Communication makes, on average, as a function of the amount of coffee she drinks before coming to work; call this relationship \( y = m(x) \).

a. What is the contextual meaning of the function value \( m(25) = 15 \)? (5 points)

On mornings when Miss Communication drinks 25 oz of coffee before coming to work, she makes, on average, 15 embarrassing typos.

b. The tangent line to \( y = m(x) \) at the point \((45, 35)\) has a slope of 2. What, including unit, is the value of \( m'(45) \)? (2 points)

\[ m'(45) = 2 \text{ typos/oz} \]

c. The numerical value of \( m''(45) \) is either 0.1 or -0.1. Which one must be correct? How do you know? (3 points)

\[ m''(45) = 0.1. \text{ } m''(45) \text{ cannot be negative because } m \text{ is concave up @ } 45. \]

d. What is the unit on \( m''(45) \)? (2 points)

\[ \frac{\text{typos/oz}}{\text{oz}} \]
7. For the function \( g(x) = (2x - 7)^5 \), the formula for the first derivative is 
\[ g'(x) = 10(2x - 7)^4. \]
State the equation of the tangent line to \( g \) at the point where \( x = 4 \).

(6 points)

A point on the line is \((4,1)\).
The slope of the line is \( g'(4) = 10 \).
The equation of the line is \( y = 10x - 39 \).

8. The function \( k' \) is shown in Figure 4. Answer each of the following questions in reference to this function; in all cases restrict your answer to values on the interval \((-7, 7)\). You do not need to explain nor do you need to answer using complete sentences. Please note - the graph is \( k' \), not \( k \). Assume that \( k \) is a continuous function.

(3 points each)

a. Over what intervals(s) is \( k \) increasing?
\((-7, 0) \text{ and } (2, 5)\)

b. At what value(s) of \( x \) is \( k \) nondifferentiable?
2

c. At what value(s) of \( x \) does an antiderivative of \( k \) have a point of inflection?
0, 2, and 5

d. Over what intervals is the function \( y = k''(x) \) constant?
\((-7, -3) \text{ and } (-3, 2)\)

e. At what value(s) of \( x \) does \( k \) have horizontal tangent lines?
0 and 5
9. Each of the following sentences is true if one of the
words/phrases in Table 1 is inserted into the blank. Find
the proper word/phrase for each of the blanks. Read each
sentence carefully!!
(2 points each)

a. If \( f' \) is positive over the entire interval \((2, 5)\), then \( f \) is ______ increasing ______ over the entire interval \((2, 5)\).

b. If the slope of \( f' \) is increasing along the entire interval \((2, 5)\), then \( f'' \) is ______ increasing ______ over the entire interval \((2, 5)\).

c. If \( g'' \) is positive over the entire interval \((2, 5)\), then \( g' \) is ______ increasing ______ over the entire interval \((2, 5)\).

d. If \( g \) is everywhere differentiable and has a local maximum at the point where \( t = 11 \),
then the tangent line to \( y = g(t) \) is ______ horizontal ______ at \( t = 11 \).

e. If the slope of \( g \) is negative over the entire interval \((2, 5)\), then \( g' \) is ______ negative ______ over the entire interval \((2, 5)\).

f. If an antiderivative of \( f \) is constantly decreasing and concave up, then \( f' \) must always be ______ positive, zero, or undefined ______.

g. If \( y = g(t) \) has a point of inflection at \( t = 7 \), then \( g''(7) \) is ______ zero or undefined ______.

h. If \( f \) has a vertical asymptote at \( x = 8 \) then \( f' \) must be ______ nondifferentiable ______ at \( x = 8 \).

Table 1: Blank Options

<table>
<thead>
<tr>
<th>Increasing</th>
<th>Decreasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concave Up</td>
<td>Concave Down</td>
</tr>
<tr>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>Zero</td>
<td>&quot;Zero or UD&quot;</td>
</tr>
<tr>
<td>(This is one choice)</td>
<td></td>
</tr>
<tr>
<td>&quot;Positive, Zero or UD&quot;</td>
<td>&quot;Negative, Zero or UD&quot;</td>
</tr>
<tr>
<td>Horizontal</td>
<td>Vertical</td>
</tr>
<tr>
<td>Nondifferentiable</td>
<td>Infinity</td>
</tr>
</tbody>
</table>

Note: UD is short for undefined