The critical numbers of \( f \) are the values of \( x \) in the domain of \( f \) where either \( f'(x) = 0 \) or \( f'(x) \) is undefined. These numbers are important because they are the only values of \( x \) where \( f \) could possibly have local extreme values.

Example 1
Consider the function \( f \) shown in Figure 1.

a. What are the critical numbers of \( f \)? Explain.

b. Where does \( f \) change concavity? What else happens at these points?

c. What are the inflection points on \( f \)? Local max points? Local min points?
Example 2

Find the critical numbers of \( g(\theta) = \frac{\ln(\theta)^2}{\theta^{1/3}} \).

Example 3

Find the critical numbers of \( f(x) = (x - 4)^4 (x + 3)^3 \).
Example 4

The first derivative of the function \( T(x) = \frac{\sqrt[3]{x-10}}{x^2} \) is \( T'(x) = \frac{-5(x-12)}{3x^3 \sqrt[3]{(x-10)^2}} \).

a. What are the critical numbers of \( T \)? Explain.

b. Build a table that clearly shows where \( T \) is increasing and where \( T \) is decreasing. Make sure your table communicates how you arrived at your conclusions.

c. What are the local maximum points on \( T \)? Local minimum points? What sort of point is \((10, 0)\)?
Example 5
Find the local minimum points, local maximum points, and inflection points on the function

\[ h(x) = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}. \]

Show all relevant work.
Example 6

Determine the intervals over which the function \( f(\theta) = -\frac{\cos^2(\theta)}{2} - \cos(\theta) - \sin(\theta) - \theta \) is increasing and the intervals over which the function is decreasing.
Example 7

Find the critical numbers of $k(t) = \frac{\sqrt{t-4}}{t-10}$.