§5.5 Graphs of the Tangent, Cotangent, Secant, and Cosecant Functions.

**Tangent Function** \( y = \tan(x) \)

\[
y = \tan(x) = \frac{\sin(x)}{\cos(x)}
\]

\( \tan(x) \) has a period of \( \pi \), so the graph has one period in the interval \((-\frac{\pi}{2}, \frac{\pi}{2})\).

\( \tan(x) \) is not defined at \( x = -\frac{\pi}{2}, \frac{\pi}{2} \) because \( \cos(-\frac{\pi}{2}) = 0 \) and \( \cos(\frac{\pi}{2}) = 0 \).

\( y = \tan(x) \) has vertical asymptotes at \( x = -\frac{\pi}{2} \) and \( x = \frac{\pi}{2} \).

**Table 5.5.1**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \tan(x) )</th>
<th>( (x,y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{\pi}{2})</td>
<td>undefined</td>
<td></td>
</tr>
<tr>
<td>(-\frac{\pi}{3})</td>
<td>(-\sqrt{3} \approx -1.73)</td>
<td>(-\frac{\pi}{3}, -\sqrt{3})</td>
</tr>
<tr>
<td>(-\frac{\pi}{4})</td>
<td>(-1)</td>
<td>(-\frac{\pi}{4}, -1)</td>
</tr>
<tr>
<td>(-\frac{\pi}{6})</td>
<td>(-\sqrt{3} \approx -0.50)</td>
<td>(-\frac{\pi}{6}, -\sqrt{3})</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>(0,0)</td>
</tr>
<tr>
<td>(\frac{\pi}{6})</td>
<td>(\sqrt{3} \approx 0.50)</td>
<td>(\frac{\pi}{6}, \sqrt{3})</td>
</tr>
<tr>
<td>(\frac{\pi}{4})</td>
<td>1</td>
<td>(\frac{\pi}{4}, 1)</td>
</tr>
<tr>
<td>(\frac{\pi}{3})</td>
<td>(\sqrt{3} \approx 1.73)</td>
<td>(\frac{\pi}{3}, \sqrt{3})</td>
</tr>
<tr>
<td>(\frac{\pi}{2})</td>
<td>undefined</td>
<td></td>
</tr>
</tbody>
</table>

\[
\lim_{x \to -\frac{\pi}{2}^+} \tan(x) = -\infty \quad \lim_{x \to -\frac{\pi}{2}^-} \tan(x) = +\infty
\]

\[
\lim_{x \to \frac{\pi}{2}^-} \tan(x) = -\infty \quad \lim_{x \to \frac{\pi}{2}^+} \tan(x) = +\infty
\]
Properties of the tangent function:
1. Domain is all real numbers except odd multiples of $\frac{\pi}{2}$.
2. The range is $(-\infty, \infty)$.
3. The tangent function is symmetric about the origin, it is an odd function.
4. The period of the tangent function is $\pi$.
5. The $x$-intercepts are
   $$\ldots, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$$
6. The vertical asymptotes are
   $$x = -3\frac{\pi}{2}, x = -\frac{\pi}{2}, x = \frac{\pi}{2}, x = 3\frac{\pi}{2}, \ldots$$
Graphing functions of the form 

\[ y = A \cdot \tan (wx) + B \]

1. Vertical stretch/shrink by a factor of \( |A| \)
   - Reflect across the x-axis if \( A < 0 \)
2. Horizontal expansion/compression by a factor of \( \frac{1}{w} \)
3. Vertical shift by \( |B| \)
   - Up if \( B > 0 \)
   - Down if \( B < 0 \)

Example: \( y = 2 \tan (\pi x) - 1 \)

1. Vertical stretch by a factor of 2
2) Horizontal Compression by a factor of $\frac{1}{7}$

period $p = \frac{\pi}{\frac{1}{7}} = 1$

$y = 2 \tan (\pi x)$

3) Vertical shift down by 1
The Graph of the Cotangent Function

\[ y = \cot(x) \]

\[ = \frac{1}{\tan(x)} \]

\( \cot(x) \) is undefined at:

\[ \cdots, -2\pi, -\pi, 0, \pi, 2\pi, \cdots \]

because \( \tan(x) = 0 \) at:

\[ \cdots, -2\pi, -\pi, 0, \pi, 2\pi, \cdots \]

\( y = \cot(x) \) has vertical asymptotes at:

\[ \cdots, -2\pi, -\pi, 0, \pi, 2\pi, \cdots \]

\( y = \tan(x) \)
5.5.6

The \( y = \cot(x) \) function has one period in the interval \([-\pi, \pi] \).

\[
\begin{align*}
\lim_{x \to 0^+} \cot(x) &= +\infty, \\
\lim_{x \to -\pi^-} \cot(x) &= -\infty
\end{align*}
\]

(a) The Graph of the Cosecant Function

\[
csc(x) = \frac{1}{\sin(x)}
\]

csc \((x)\) is undefined at \(-2\pi, -\pi, 0, \pi, 2\pi, \ldots\)

because \(\sin(x) = 0\) at \(-\pi, 0, \pi, 2\pi, \ldots\)

\(y = \csc(x)\) has vertical asymptotes at \(x = -2\pi, x = -\pi, x = 0, x = \pi, x = 2\pi, \ldots\)

(b) The graph of the Secant Function:

\[
\sec(x) = \frac{1}{\cos(x)}
\]

is undefined at \(\ldots, -3\pi, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\)

since \(\cos(x) = 0\) at \(\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\)

\(y = \sec(x)\) has vertical asymptotes at \(\ldots, x = -\frac{3\pi}{2}, x = -\frac{\pi}{2}, x = \frac{\pi}{2}, x = \frac{3\pi}{2}, \ldots\)
Example: page 442
(30) Graph the function \( y = \csc\left(\frac{3\pi}{2}x\right) \)

Period: \( p = \frac{\frac{3\pi}{2}}{\frac{3\pi}{2}} = \frac{2\pi}{\frac{3\pi}{2}} = \frac{4}{3} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \csc\left(\frac{3\pi}{2}x\right) = \frac{1}{\sin(\frac{3\pi}{2}x)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 )</td>
<td>( \csc(0) ) is undefined</td>
</tr>
<tr>
<td>( \frac{1}{3} )</td>
<td>( \csc\left(\frac{\pi}{3}\right) = 1 )</td>
</tr>
<tr>
<td>( \frac{2}{3} )</td>
<td>( \csc(\pi) ) is undefined</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( \csc\left(\frac{3\pi}{2}\right) = -1 )</td>
</tr>
<tr>
<td>( \frac{4}{3} )</td>
<td>( \csc(2\pi) ) is undefined</td>
</tr>
</tbody>
</table>

\( y = \csc\left(\frac{3\pi}{2}x\right) \)
Example: page 443

(a) Carrying a ladder around a corner.

Two hallways, one with width 4 feet, the other with width 3 feet, meet at a right angle.

(b) Show that the length, \( L \), of the line segment shown, as a function of the angle \( \theta \), is

\[ L(\theta) = 3 \cdot \sec(\theta) + 4 \cdot \csc(\theta) \]

\[
\begin{align*}
A & \quad \theta & \quad e_1 & \quad e_2 \\
3 & \quad 4 & \quad 3 & \quad 4 & \quad 3 & \quad 4 \\
\end{align*}
\]

\[
L = e_1 + e_2
\]

\[
A. \quad \cos(\theta) = \frac{3}{e_1} \quad \Rightarrow \quad e_1 = \frac{3}{\cos(\theta)} = 3 \cdot \sec(\theta)
\]

\[
B. \quad \sin(\theta) = \frac{4}{e_2} \quad \Rightarrow \quad e_2 = \frac{4}{\sin(\theta)} = 4 \cdot \csc(\theta)
\]

\[
L = e_1 + e_2
\]

\[
L = 3 \cdot \sec(\theta) + 4 \cdot \csc(\theta)
\]

(b) Graph \( L = 3 \cdot \sec(\theta) + 4 \cdot \csc(\theta) \) for \( 0 \leq \theta \leq \frac{\pi}{2} \).
For what value of $\theta$ is $L$ least?

The smallest value of $L$ is

$L \approx 9.86$ feet when $\theta \approx 0.8333$ (radians).

What is the longest ladder that can be carried around the corner?

$L \approx 9.86$ feet

as you carry the ladder around the corner. $\theta$ takes every value between 0 and $\frac{\pi}{2}$ (radians)

so it can be no larger than the smallest value of $L$ for $0 \leq \theta \leq \frac{\pi}{2}$. 