5.3.1

85.3 Properties of Trigonometric Functions

Let $\theta$ be an angle in standard position, and let $P = (x, y)$ be the point on the unit circle determined by $\theta$, then:

- $\sin(\theta) = y$
- $\cos(\theta) = x$
- $\tan(\theta) = \frac{y}{x}$ if $x \neq 0$
- $\csc(\theta) = \frac{1}{y}$ if $y \neq 0$
- $\sec(\theta) = \frac{1}{x}$ if $x \neq 0$
- $\cot(\theta) = \frac{x}{y}$ if $y \neq 0$

**Domain**

- $\sin(\theta)$ and $\cos(\theta)$ are defined for every real number $\theta$, their domain is $(-\infty, \infty)$.
- $\tan(\theta)$ and $\sec(\theta)$ are undefined whenever $x = 0$
- $x = 0$ whenever $P$ is on the $y$-axis, i.e. $P = (0, 1)$ at $\frac{\pi}{2}$ (90°) and $P = (0, -1)$ at $\frac{3\pi}{2}$ (270°)

More generally, $\tan(\theta)$ and $\sec(\theta)$ are undefined at any angles that are coterminal with $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, that is at any odd multiple of $\frac{\pi}{2}$. 
5.3.2

\[ \tan(\theta) \] and \( \sec(\theta) \) are undefined at
\[ \pm \frac{\pi}{2} \pm \frac{2\pi}{2} \pm \frac{3\pi}{2} \pm \frac{4\pi}{2} \pm \frac{5\pi}{2} \pm \frac{6\pi}{2} \]

The domain of \( \tan(\theta) \) and \( \sec(\theta) \)
\[ (-\frac{5\pi}{2}, -\frac{3\pi}{2}) \cup (-\frac{\pi}{2}, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2}) \cup ...

\( \csc(\theta) \) and \( \cot(\theta) \) are undefined whenever \( y = 0 \) i.e.
\[ p = (0, 1) \pm 0 \pm \pi \pm 2\pi \pm ...
\csc(\theta) \) and \( \cot(\theta) \) are undefined at 0 and \( \pi \).

More generally, they are undefined at any angles that are co-linear with 0 or \( \pi \) that is at any integer multiple of \( \pi \)
\[ (-\frac{3\pi}{2}, -\frac{\pi}{2}) \cup (-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2}) \cup ...

The domain of \( \csc(\theta) \) and \( \cot(\theta) \)
\[ (-3\pi, -2\pi) \cup (-2\pi, -\pi) \cup (-\pi, 0) \cup (0, \pi) \cup (\pi, 2\pi) \cup (2\pi, 3\pi) \cup ... \]
- **Ranges**

If $\theta$ is an angle in standard position and $P = (x, y)$ is the point on the unit circle determined by $\theta$, then $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ since $\sin(\theta) = y$ and $\cos(\theta) = x$.

So the range of the sine function and the cosine function is $[-1, 1]$.

$tan(\theta) = \frac{y}{x}$ and $cot(\theta) = \frac{x}{y}$

when $x$ is close to 0, $y$ is either close to 1 or close to -1.

So $tan(\theta)$ takes very large positive or negative values when $x$ is close to 0 but $x \neq 0$.

So the range of $tan(\theta)$ is all real numbers $= (-\infty, \infty)$.

when $y$ is close to 0, $x$ is either close to 1 or close to -1.

So $cot(\theta)$ takes very large positive or negative values when $y$ is close to 0.

So the range of $cot(\theta)$ is all real numbers $= (-\infty, \infty)$. 

\[ \csc (\theta) = \frac{1}{y} \quad \text{if } y \neq 0 \]
\[ \text{if } -1 \leq y < 0 \text{ then } -\infty < \frac{1}{y} \leq -1 \]
\[ \text{so } \csc (\theta) \text{ belongs to } (-\infty, -1] \]
\[ \text{if } 0 < y \leq 1 \text{ then } 1 \leq \frac{1}{y} < \infty \]
\[ \text{so } \csc (\theta) \text{ belongs to } [1, \infty) \]
\[ \text{The range of } \csc (\theta) \text{ is } (-\infty, -1] \cup [1, \infty) \]

Similarly,
\[ \sec (\theta) = \frac{1}{x} \quad \text{so the range of } \sec (\theta) = (-\infty, -1] \cup [1, \infty) \]

---

**Period of Trigonometric Functions**

Two different angles are called co-terminal if they have the same terminal side.

If two angles are co-terminal, they determine the same point \( P = (x, y) \) on the unit circle, so they have exactly the same values for all of the trigonometric functions.

Two angles are co-terminal if and only if they differ by an integer multiple of \( 2\pi \) (or \( 360^\circ \)) so by some number of complete revolutions either in the counter-clockwise (positive) direction or the clockwise (negative) direction.

Example: \( 0 \) and \( 2\pi \) are co-terminal,
\[ \frac{\pi}{2} \text{ and } \frac{3\pi}{2} \text{ are co-terminal}, \]
\[ -\pi \text{ and } \pi \text{ are co-terminal}, \]
\[ -\frac{\pi}{4} \text{ and } \frac{9\pi}{4} \text{ are co-terminal}, \]
\[ -\frac{\pi}{2} \text{ and } \frac{3\pi}{2} \text{ are co-terminal}. \]
If $\theta$ is any angle then $\theta$ is coterminal with any angle of the form $\theta + 2\pi k$ where $k$ is a (positive or negative) integer. Hence:

$\sin(\theta + 2\pi k) = \sin(\theta) \quad \text{for any } k$

$\cos(\theta + 2\pi k) = \cos(\theta) \quad \text{for any } k$

$csc(\theta + 2\pi k) = csc(\theta) \quad \text{for any } k$

$sec(\theta + 2\pi k) = sec(\theta) \quad \text{for any } k$

It is also true that

$tan(\theta + 2\pi k) = tan(\theta) \quad \text{for any } k$

$cot(\theta + 2\pi k) = cot(\theta) \quad \text{for any } k$

But if $P(x_1,y_1)$ is the point determined by $\theta$ then the point determined by $\theta + k\pi$ is $P(-x_1,-y_1)$ whenever $k$ is an odd integer.

And

$\tan(\theta) = \frac{y}{x}$

so

$\tan(\theta + k\pi) = \frac{-y}{-x} = \frac{y}{x} = \tan(\theta)$

Al

$\cot(\theta) = \frac{x}{y}$

so

$\cot(\theta + k\pi) = \frac{x}{y} = \frac{x}{-y} = -\cot(\theta)$

$\tan(\theta + k\pi) = \tan(\theta)$

$\cot(\theta + k\pi) = \cot(\theta)$ \text{ whenever } k \text{ is an odd integer.}
5.3.6

**Definitions:** A function \( f \) is called periodic if and only if there is a positive number \( P \) such that \( f(\theta + P) = f(\theta) \) for all \( \theta \) in the domain of \( f \).

If there is a smallest such number \( P \), this smallest value is called the fundamental period of \( f \).

- \( \sin(\theta) \), \( \cos(\theta) \), \( \csc(\theta) \), \( \cot(\theta) \)
- All have a fundamental period of \( 2\pi \).
- \( \tan(\theta) \) and \( \cot(\theta) \) both have a fundamental period of \( \pi \).

**Example:** Use the fact that the six trigonometric functions are periodic to find the values of each expression.

1. \( \cos(420^\circ) \):
   - \( 420^\circ = 60^\circ + 360^\circ \)
   - \( 420^\circ \) is equivalent to \( 60^\circ \).
   - \( \cos(420^\circ) = \cos(60^\circ) = \frac{1}{2} \)

2. \( \sin\left(\frac{9\pi}{4}\right) \):
   - \( \frac{9\pi}{4} = 2\pi + \frac{\pi}{4} \)
   - \( \frac{9\pi}{4} \) is coterminal with \( \frac{\pi}{4} \).
   - \( \sin\left(\frac{9\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \).
\[ \text{sec} \left( \frac{2 \pi}{3} \right) \]

\[ \frac{2 \pi}{3} = 4 + \frac{1}{2} \Rightarrow \frac{2 \pi}{3} = 4 \pi + \frac{7}{6} \]

\[ \Rightarrow \frac{2 \pi}{3} \text{ is co-terminal with } 4 \pi \]

The point determined by \( \frac{2 \pi}{3} \) is \( P = \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \)

\[ x = \frac{\sqrt{3}}{2} \Rightarrow \text{sec} \left( \frac{2 \pi}{3} \right) = \text{sec} \left( \frac{\pi}{6} \right) = \frac{1}{x} \]

\[ \Rightarrow \text{sec} \left( \frac{2 \pi}{3} \right) = \frac{1}{\frac{\sqrt{3}}{2}} \]

\[ = \frac{2}{\sqrt{3}} \]

\[ = \frac{2 \sqrt{3}}{3} \]

\[ \Rightarrow \text{Determine the sign of the trig functions in each of the four quadrants} \]

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>Sin(( \theta ))</th>
<th>Csc(( \theta ))</th>
<th>Csc(( \theta ))</th>
<th>Sec(( \theta ))</th>
<th>Cot(( \theta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>QI</td>
<td>Positive</td>
<td>Positive</td>
<td>Positive</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td>QII</td>
<td>Positive</td>
<td>Negative</td>
<td>Positive</td>
<td>Negative</td>
<td>Negative</td>
</tr>
<tr>
<td>QIII</td>
<td>Negative</td>
<td>Negative</td>
<td>Negative</td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>QIV</td>
<td>Negative</td>
<td>Positive</td>
<td>Positive</td>
<td>Negative</td>
<td>Negative</td>
</tr>
</tbody>
</table>
Page 418. Find the quadrant in which θ lies:

23. \( \sin(θ) < 0, \cos(θ) > 0 \)  \( x > 0, y < 0 \); Q IV
24. \( \cos(θ) < 0, \tan(θ) > 0 \)  \( x < 0, y > 0 \); Q III

- Fundamental Identities of Trig Functions:
  If θ is an angle in standard position and \( P = (x, y) \) is the point determined by θ then:
  \( \sin(θ) = y \) and \( \csc(θ) = \frac{1}{y} \) if \( y \neq 0 \)
  Therefore \( \csc(θ) = \frac{1}{\sin(θ)} \)
  whenever \( \csc(θ) \) is defined.

- \( \cos(θ) = x \) and \( \sec(θ) = \frac{1}{x} \) if \( x \neq 0 \)
  Therefore \( \sec(θ) = \frac{1}{\cos(θ)} \)
  whenever \( \sec(θ) \) is defined.

- \( \tan(θ) = \frac{x}{y} \) whenever \( y \neq 0 \)
  \( \cot(θ) = \frac{1}{\tan(θ)} \)
  whenever \( \cot(θ) \) is defined.

- Reciprocal Identities:
  \( \csc(θ) = \frac{1}{\sin(θ)} \)
  \( \sec(θ) = \frac{1}{\cos(θ)} \)
  \( \cot(θ) = \frac{1}{\tan(θ)} \)
Also:
\[ \sin(\theta) = y \quad \text{and} \quad \cos(\theta) = x \]
so
\[ \tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)} \]
and
\[ \cot(\theta) = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)} \]

Reciprocal Identities:
\[ \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \text{and} \quad \cos(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \]

Notice: All of the Trig Functions can be written in terms of \( \sin(\theta) \) and \( \cos(\theta) \)

- \( \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \)
- \( \csc(\theta) = \frac{1}{\sin(\theta)} \)
- \( \sec(\theta) = \frac{1}{\cos(\theta)} \)
- \( \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \)

Pythagorean Identities
The equation of the unit circle is \( x^2 + y^2 = 1 \)
or equivalently
\[ y^2 + x^2 = 1 \]
5.3.10

If $\theta$ is an angle in standard position and $P = (x,y)$ is the point on the unit circle determined by $\theta$

then $\cos(\theta) = x$ and $\sin(\theta) = y$

since $y^2 + x^2 = 1$ it follows

$(\sin(\theta))^2 + (\cos(\theta))^2 = 1$

This is usually written as

$\sin^2(\theta) + \cos^2(\theta) = 1$

If we divide the equation through by $\cos^2(\theta)$ we obtain:

$\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$

or equivalently:

$\left(\frac{\sin(\theta)}{\cos(\theta)}\right)^2 + 1 = \left(\frac{1}{\cos(\theta)}\right)^2$

or equivalently:

$(\tan(\theta))^2 + 1 = (\sec(\theta))^2$

Which is usually written as:

$\tan^2(\theta) + 1 = \sec^2(\theta)$
B. 3. 11

Starting with:

\[ \sin^2(\theta) + \cos^2(\theta) = 1 \]

dividing through by \( \sin^2(\theta) \)
we obtain:

\[ \frac{\sin^2(\theta)}{\sin^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)} \]

equivalently:

\[ 1 + \left( \frac{\cos(\theta)}{\sin(\theta)} \right)^2 = \left( \frac{1}{\sin(\theta)} \right)^2 \]

equivalently:

\[ 1 + \cot^2(\theta) = \csc^2(\theta) \]

which is usually written as:

\[ 1 + \cot^2(\theta) = \csc^2(\theta) \]

Pythagorean Identities

- \( \sin^2(\theta) + \cos^2(\theta) = 1 \)
- \( \tan^2(\theta) + 1 = \sec^2(\theta) \)
- \( 1 + \cot^2(\theta) = \csc^2(\theta) \)
Fundamental Identities

- Reciprocal Identities:
  \[
  \csc(\theta) = \frac{1}{\sin(\theta)}, \quad \sec(\theta) = \frac{1}{\cos(\theta)}, \quad \cot(\theta) = \frac{1}{\tan(\theta)}
  \]

- Quotient Identities:
  \[
  \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}, \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}
  \]

- Pythagorean Identities:
  \[
  \sin^2(\theta) + \cos^2(\theta) = 1, \\
  \tan^2(\theta) + 1 = \sec^2(\theta), \\
  1 + \cot^2(\theta) = \csc^2(\theta)
  \]

Example: Use Identities to evaluate the expression:

\[
1 + \frac{\cos^2(\frac{\pi}{3})}{\sin^2(\frac{\pi}{3})} = 1 + \cot^2(\frac{\pi}{3}) = \csc^2(\frac{\pi}{3}) = \left[\frac{1}{\sin(\frac{\pi}{3})}\right]^2 = \left[\frac{\sqrt{3}}{2}\right]^2 = \left[\frac{2}{\sqrt{3}}\right]^2 = \frac{4}{3}.
\]
Note:

For any angle \( \Theta \)

\[
\sin^2(\Theta) + \cos^2(\Theta) = 1 \implies \\
\sin^2(\Theta) = 1 - \cos^2(\Theta) \implies \\
\sin(\Theta) = \pm \sqrt{1 - \cos^2(\Theta)}
\]

The + or - is determined by the quadrant of the angle.

Similarly

\[
\cos^2(\Theta) = 1 - \sin^2(\Theta) \implies \\
\cos(\Theta) = \pm \sqrt{1 - \sin^2(\Theta)}
\]

The + or - is determined by the quadrant of the angle.

Example: page 418

Find the values of all 6 trig functions if

\( \cos(\Theta) = \frac{3}{5} \) and \( \Theta \) is in QIV in Q IV \( \cos(\Theta) > 0; \sin(\Theta) < 0 \)

\[
\sin(\Theta) = -\sqrt{1 - \cos^2(\Theta)} \\
= -\sqrt{1 - \left(\frac{3}{5}\right)^2} \\
= -\sqrt{1 - \frac{9}{25}} \\
= -\sqrt{\frac{25}{25} - \frac{9}{25}} \\
= -\sqrt{\frac{16}{25}} \\
= -\frac{4}{5}
\]

\( \tan(\Theta) = -\frac{4}{3} \)
\[ \sin(\theta) = -\frac{4}{5} \quad \cos(\theta) = \frac{3}{5} \]

\[ \tan(\theta) = -\frac{\sin(\theta)}{\cos(\theta)} = -\frac{4}{3} \]

\[ \csc(\theta) = \frac{1}{\sin(\theta)} = -\frac{5}{4} \]

\[ \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{5}{3} \]

\[ \cot(\theta) = \frac{1}{\tan(\theta)} = -\frac{3}{4} \]
\[ \text{5.3.15} \]

Page 418

(54) \( \csc(\theta) = 3 \) if \( \cot(\theta) < 0 \)
\( \csc(\theta) > 0 \Rightarrow \sin(\theta) > 0 \Rightarrow \text{Q I or Q II} \)
\( \cot(\theta) < 0 \Rightarrow \cos(\theta) < 0 \Rightarrow \text{Q II or Q III} \)

hence the angle is in Q III
\( \csc(\theta) = 3 \Rightarrow \frac{1}{\sin(\theta)} = 3 \Rightarrow \sin(\theta) = \frac{1}{3} \)

* \( \sin(\theta) = \frac{\sqrt{3}}{3} \)
* \( \cos(\theta) < 0 \Rightarrow \frac{1}{\cos(\theta)} = -\sqrt{1 - \sin^2(\theta)} \\
\cos(\theta) = -\sqrt{1 - \left(\frac{\sqrt{3}}{3}\right)^2} \\
= -\sqrt{1 - \left(\frac{1}{3}\right)^2} \\
= -\sqrt{1 - \frac{1}{9}} \\
= -\sqrt{\frac{8}{9}} \\
= -\sqrt{\frac{8}{3}} \\
= -\frac{2\sqrt{2}}{3} \)
* \( \cos(\theta) = -2\frac{\sqrt{2}}{3} \)

* \( \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \\
= \frac{\sqrt{3}}{-2\sqrt{2}}/3 \\
= \frac{1}{3} \cdot \frac{3}{-2\sqrt{2}} \\
= -\frac{1}{2\sqrt{2}} \\
= -\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\
= -\frac{\sqrt{2}}{4} \)
\[ \csc(\theta) = 3 \] (given)

\[ \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{-3}{2\sqrt{2}} \]
\[ = -\frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \]
\[ = -\frac{3\sqrt{2}}{4} \]

\[ \cot(\theta) = \frac{1}{\tan(\theta)} = -2\sqrt{2} \]
5.3.17

**Even and Odd Properties**

A function \( f \) is **even** if and only if \( f(-x) = f(x) \) for all \( x \) in the domain of \( f \).

A function \( f \) is **odd** if and only if \( f(-x) = -f(x) \) for all \( x \) in the domain of \( f \).

\[
P = (x, y)
\]

\[
P(x, y)
\]

- \( \cos(-\theta) = x = \cos(\theta) \)
- \( \sin(-\theta) = -y = -\sin(\theta) \)

- \( \cos(\theta) \) is an **even** function
- \( \sin(\theta) \) is an **odd** function

- \( \tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} \)
  \[
  = -\frac{\sin(\theta)}{\cos(\theta)}
  = -\tan(\theta)
  \]
  \( \Rightarrow \) \( \tan(\theta) \) is an **odd** function

- \( \csc(-\theta) = \frac{1}{\sin(-\theta)} \)
  \[
  = -\frac{1}{\sin(\theta)}
  = -\csc(\theta)
  \]
  \( \Rightarrow \) \( \csc(\theta) \) is an **odd** function
\[
\sec(\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)} = \sec(\theta)
\]
\[
\Rightarrow \sec(\theta) \text{ is an even function}
\]
\[
\cot(-\theta) = \frac{1}{\tan(-\theta)} = \frac{1}{-\tan(\theta)} = -\cot(\theta)
\]
\[
\Rightarrow \cot(\theta) \text{ is an odd function}
\]

- \(\sin(\theta)\) odd
- \(\cos(\theta)\) even
- \(\tan(\theta)\) odd
- \(\csc(\theta)\) odd
- \(\sec(\theta)\) even
- \(\cot(\theta)\) odd

Examples page 419

Find the exact values using even/odd properties:

1. \(\cos(-30^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}\)
2. \(\sin(-135^\circ) = -\sin(135^\circ) = -\frac{\sqrt{2}}{2}\)
3. \(\sin(-\pi) = -\sin(\pi) = 0\)
4. \(\sin(-\frac{\pi}{3}) = -\sin(\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}\)
5. 3. 10

P. 410

Use properties of trig functions to find the exact values (don't use a calculator)

78) \( \sec^2(18^\circ) - \tan^2(18^\circ) \)

\[
\tan^2(18^\circ) + 1 = \sec^2(18^\circ) \]

So

\[
\sec^2(18^\circ) - \tan^2(18^\circ) = \tan^2(18^\circ) + 1 - \tan^2(18^\circ) = 1
\]

80) \( \tan(10^\circ) \cot(10^\circ) = \frac{1}{\tan(10^\circ)} \cot(10^\circ) = 1 \)

82) \( \cot(20^\circ) - \frac{\cos(20^\circ)}{\sin(20^\circ)} = \cot(20^\circ) - \cot(20^\circ) = 0 \)

86) \( \sec(\pi/18) \cdot \cos(37\pi/18) = \sec(\pi/18) \cdot \cos(\pi/18 + 3(\pi/18)) = \sec(\pi/18) \cdot \cos(\pi/18 + 2\pi) = \left(\frac{1}{\cos(\pi/18)}\right) \cdot \cos(\pi/18) = 1 \)
5.3.20

**94** Find the exact value of
\[
\cos(1^\circ) + \cos(2^\circ) + \cos(3^\circ) + \cdots + \cos(180^\circ) + \cos(360^\circ)
\]

**Notice:**
\[
\cos(\theta + 180^\circ) = -\cos(\theta)
\]

The point determined by \(\theta\) is \(P = (x, y)\)

The point determined by \(\theta + \pi\) is \(P = (-x, -y)\)

**hence** \(\cos(\theta + \pi) = -x = -\cos(\theta)\)

so:
\[
\begin{align*}
\cos(181^\circ) &= -\cos(1^\circ) \\
\cos(182^\circ) &= -\cos(2^\circ) \\
\cos(183^\circ) &= -\cos(3^\circ) \\
\vdots \\
\cos(360^\circ) &= -\cos(179^\circ)
\end{align*}
\]

\[
\begin{align*}
\cos(4^\circ) + \cos(2^\circ) + \cos(3^\circ) + \cdots + \cos(360^\circ) &= \\
[\cos(1^\circ) + \cos(181^\circ)] + [\cos(2^\circ) + \cos(182^\circ)] + \\
[\cos(3^\circ) + \cos(183^\circ)] + \cdots + [\cos(179^\circ) + \cos(180^\circ)] + \cos(180^\circ) \\
= 0 + 0 + 0 + \cdots + 0 + \cos(180^\circ) \\
= 0 + (-1) \\
= -1
\end{align*}
\]