Answers to the problems to help you review for Test 2

Solve the following equations algebraically. Check your solutions. Write your answers in complete sentences.

1. \[6x - 9 = 21\]
   \[6x - 9 = 21\]
   \[6x - 9 + 9 = 21 + 9\]
   \[6x = 30\]
   \[\frac{6x}{6} = \frac{30}{6}\]
   \[x = 5\]
   The check is left as an exercise.

   The solution is 5.

2. \[-8 + 3A = 4A - 5 + 2A - 2\]
   \[-8 + 3A = 4A - 5 + 2A - 2\]
   \[-8 + 3A = 6A - 7\]
   \[-8 + 3A + 7 = 6A - 7 + 7\]
   \[-1 + 3A - 3A = 6A - 3A\]
   \[-1 = 3A\]
   \[3 = \frac{3A}{3}\]
   The check is left as an exercise.

   The solution is \(-\frac{1}{3}\).

3. \[3y - 7(y + 4) = -y + 6 - 3y\]
   \[3y - 7(y + 4) = -y + 6 - 3y\]
   \[3y - 7y - 28 = -4y + 6\]
   \[-4y - 28 = -4y + 6\]
   \[-4y - 28 + 4y = -4y + 6 + 4y\]
   \[-28 = -6\]
   This is a contradiction.

   There is no solution.

4. \[\frac{1}{2}(4p - 6) = -3p + 2\]
   \[\frac{1}{2}(4p - 6) = -3p + 2\]
   \[2 \cdot \frac{1}{2}(4p - 6) = 2(-3p + 2)\]
   \[4p - 6 = -6p + 4\]
   \[4p - 6 + 6 = -6p + 4 + 6\]
   \[4p = -6p + 10\]
   \[4p + 6p = -6p + 10 + 6p\]
   \[10p = 10\]
   \[\frac{10p}{10} = \frac{10}{10}\]
   \[p = 1\]
   The check is left as an exercise.

   The solution is 1.
5. \( -(3z + 8) - 9 = 17 + 3z \)

\[-(3z + 8) - 9 = 17 + 3z \]
\[-3z - 8 - 9 = 17 + 3z \]
\[-3z - 17 = 17 + 3z \]
\[-3z - 17 + 3z = 17 + 3z + 3z \]
\[-17 = 17 + 6z \]
\[-17 - 17 = 17 + 6z - 17 \]
\[-34 = 6z \]
\[-34 \div 6 = 6z \]
\[-17 \div \frac{2}{3} = z \]
\[z = -\frac{17}{3} \]

The check is left as an exercise.

The solution is \(-\frac{17}{3}\).

6. \( 15 - \frac{2b}{3} = \left( -\frac{4b}{6} + 8 \right) + 7 \)

\[15 - \frac{2b}{3} = \left( -\frac{4b}{6} + 8 \right) + 7 \]
\[15 - \frac{2b}{3} = -\frac{4b}{6} + 15 \]
\[6 \left( 15 - \frac{2b}{3} \right) = 6 \left( -\frac{4b}{6} + 15 \right) \]
\[90 - \frac{12b}{3} = -\frac{24b}{6} + 90 \]
\[90 - 4b + 4b = -4b + 90 + 4b \]
\[90 = 90 \]

This is an identity.

Every real number is a solution.

7. \( \frac{3r}{5} - \frac{2}{5} = \frac{r}{3} + \frac{2}{5} \)

\[\frac{3r}{5} - \frac{2}{5} = \frac{r}{3} + \frac{2}{5} \]
\[15 \left( \frac{3r}{5} - \frac{2}{5} \right) = 15 \left( \frac{r}{3} + \frac{2}{5} \right) \]
\[\frac{45r}{5} - \frac{30}{5} = \frac{15r}{3} + \frac{30}{5} \]
\[9r - 6 + 6 = 5r + 6 + 6 \]
\[9r = 5r + 12 \]
\[9r - 5r = 5r + 12 - 5r \]
\[4r = 12 \]
\[\frac{4r}{4} = \frac{12}{4} \]
\[r = 3 \]

The solution is 3.
Solve each inequality. Express the solution in interval notation. Graph the solution set on a number line. If the inequality has no solution or is true for all real numbers, so state.

8. \(-3d < 12\)

\[-3d < 12\]
\[-3d \quad 12\]
\[\frac{-3}{-3} > \frac{-12}{-3}\]
\[d > -4\]
\[(-4, \infty)\]

9. \(7x - 4 \geq 3x\)

\[7x - 4 \geq 3x\]
\[7x - 4 - 3x \geq 3x - 3x\]
\[4x - 4 \geq 0\]
\[4x - 4 + 4 \geq 0 + 4\]
\[4x \geq 4\]
\[\frac{4x}{4} \geq \frac{4}{4}\]
\[x \geq 1\]
\[[1, \infty)\]

10. \(2m + 8 \leq 4m - 2(m + 5)\)

\[2m + 8 \leq 4m - 2m - 10\]
\[2m + 8 \leq 2m - 10\]
\[2m + 8 - 2m \leq 2m - 10 - 2m\]
\[8 \leq -10\]
This is a contradiction.
There is no solution to this inequality.

11. \((-2q + 4) < -1 + 3q\)

\[13 - (2q + 4) < -1 + 3q\]
\[13 - 2q - 4 < -1 + 3q\]
\[9 - 2q < -1 + 3q\]
\[9 - 2q + 2q < -1 + 3q + 2q\]
\[9 < -1 + 5q\]
\[9 + 1 < -1 + 5q + 1\]
\[10 < 5q\]
\[\frac{10}{5} < \frac{5q}{5}\]
\[2 < q\]
\[q > 2\]
\[(2, \infty)\]
12. \(4 - w + 3(5 + w) < 25\)

\[
4 - w + 3(5 + w) < 25 \\
4 - w + 15 + 3w < 25 \\
2w + 19 < 25 \\
2w + 19 - 19 < 25 - 19 \\
2w < 6 \\
\frac{2w}{2} < \frac{6}{2} \\
w < 3 \\
(-\infty, 3)
\]

13. Use interval notation to describe the following graph.

(\(-3, \infty\))

14. Solve \(x + 2y = 6\) for \(y\).

\[
x + 2y = 6 \\
x + 2y - x = 6 - x \\
2y = 6 - x \\
\frac{2y}{2} = \frac{6 - x}{2} \\
y = \frac{6 - x}{2} \quad \text{which can also be written} \quad y = \frac{-x}{2} + 3
\]
15. Solve \( A = P + Prt \) for \( t \).

\[
\begin{align*}
A &= P + Prt \\
A - P &= P + Prt - P \\
A - P &= Prt \\
A - P &= Pr \\
\frac{A - P}{Pr} &= \frac{Prt}{Pr} \\
\frac{A - P}{Pr} &= t \\
t &= \frac{A - P}{Pr}
\end{align*}
\]

16. Solve \( A = P + Prt \) for \( P \).

\[
\begin{align*}
A &= P + Prt \\
A &= P(1 + rt) \\
\frac{A}{1 + rt} &= \frac{P(1 + rt)}{1 + rt} \\
\frac{A}{1 + rt} &= P \\
P &= \frac{A}{1 + rt}
\end{align*}
\]

17. What is 136\% of 700?

\[
\begin{align*}
A &= PB \\
A &= 1.36(700) \\
\quad &= 952 \\
952 \text{ is 136\% of 700.}
\end{align*}
\]

18. 42 is what percent of 672?

\[
\begin{align*}
A &= PB \\
42 &= P(672) \\
\frac{42}{672} &= \frac{672P}{672} \\
0.0625 &= P \\
6.25\% &= P \\
42 \text{ is 6.25\% of 672.}
\end{align*}
\]
19. Solve the proportion \[ \frac{3}{n+3} = \frac{9}{24} \]

\[ \frac{3}{n+3} = \frac{9}{24} \]

\[ 3(24) = 9(n + 3) \]

\[ 72 = 9n + 27 \]

\[ 72 - 27 = 9n + 27 - 27 \]

\[ 45 = 9n \]

\[ \frac{45}{9} = \frac{9n}{9} \]

\[ 5 = n \]

The solution is 5.

For the following problems, define your variable (include units), set up an algebraic equation to solve the problem, use algebra to solve the problem, and answer the question in a complete sentence.

20. The sum of a number and 17 is equal to twice the number. Find the number.

Let \( n \) represent the number.

\[ n + 17 = 2n \]

\[ n + 17 - n = 2n - n \]

\[ 17 = n \]

The number is 17.

21. The sales tax is 6%. If the total cost of the book (including tax) is $26.50, what is the price of the book before the tax?

Let \( P \) represent the price (in dollars) of the book before the tax.

price + sales tax(price) = total cost

\[ P + 0.06P = 26.50 \]

\[ 1.06P = 26.50 \]

\[ \frac{1.06P}{1.06} = \frac{26.50}{1.06} \]

\[ P = 25 \]

The price of the book before the tax was $25.
22. One cookie jar contains sugar cookies and another jar contains chocolate chip cookies. If there are two dozen more sugar cookies than chocolate chip cookies and 96 cookies in total, how many of each type of cookie are there?

Let $N$ represent the number of chocolate chip cookies. Then $N+24$ represents the number of sugar cookies.

\[
N + N + 24 = 96
\]

\[
2N + 24 = 96
\]

\[
2N + 24 - 24 = 96 - 24
\]

\[
2N = 72
\]

\[
\frac{2N}{2} = \frac{72}{2}
\]

\[
N = 36
\]

So $N + 24 = 60$

There are 60 sugar cookies and 36 chocolate chip cookies.

23. A student has made a 79, 56, 91, and 72 on four major exams. Determine the possible scores on the fifth exam that will result in a 75 or higher average.

Let $G$ represent the score the student earns on the fifth exam.

\[
\frac{79 + 56 + 91 + 72 + G}{5} \geq 75
\]

\[
\frac{298 + G}{5} \geq 75
\]

\[
5 \cdot \frac{298 + G}{5} \geq 5 \cdot 75
\]

\[
298 + G \geq 375
\]

\[
298 + G - 298 \geq 375 - 298
\]

\[
G \geq 77
\]

The student would need to score 77 or higher.
24. A college student has budgeted $25 a month for phone service. If the telephone company charges $18 a month for basic service and $0.05 a minute for each long distance call, how many minutes can be spent on long distance calls each month?

Let \( M \) represent the number of minutes the student can use for long distance calls each month.

Total budgeted = base cost + (cost per minute)(number of minutes)

\[
25 = 18 + 0.05M
\]

\[
25 - 18 = 18 + 0.05M - 18
\]

\[
7 = 0.05M
\]

\[
\frac{7}{0.05} = \frac{0.05M}{0.05}
\]

\[
140 = M
\]

The student can use up to 140 minutes for long distance calls.