Section IV: Radical Expressions, Equations, and Functions

Module 3: Multiplying Radical Expressions

Recall the property of exponents that states that $a^mb^m = (ab)^m$. We can use this rule to obtain an analogous rule for radicals:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}}$$

$$= (ab)^{\frac{1}{n}} \quad \text{(using the property of exponents given above)}$$

$$= \sqrt[n]{ab}$$

**Product Rule for Radicals**

If $a$ and $b$ are positive real numbers and $n$ is a positive integer, then $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

**EXAMPLE:** Perform the indicated multiplication, and simplify completely.

a. $\sqrt{2} \cdot \sqrt{18}$

b. $\sqrt[4]{3x^2} \cdot \sqrt[4]{27x^2}$

**SOLUTIONS:**

a. $\sqrt{2} \cdot \sqrt{18} = \sqrt{2 \cdot 18} = \sqrt{36} = 6$ (product rule for radicals, write 36 as a perfect square)

b. $\sqrt[4]{3x^2} \cdot \sqrt[4]{27x^2} = \sqrt[4]{3x^2 \cdot 27x^2} = \sqrt[4]{81x^4}$ (product rule for radicals)

$$= \sqrt[4]{81} \cdot \sqrt[4]{x^4}$$

$$= 3|x| \quad \text{(we need to use the absolute value since 4 is even)}$$
**Product Rule for Simplifying Radical Expressions:**

When simplifying a radical expression it is often necessary to use the following equation which is equivalent to the product rule:

\[ \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} . \]

**EXAMPLE:** Simplify \( \sqrt{40} \).

**SOLUTION:** Since 40 isn't a perfect square, we need to write 40 as a product containing a factor that is a perfect square:

\[
\sqrt{40} = \sqrt{4 \cdot 10} \quad \text{(factor 40 using perfect square(s))}
\]
\[
= \sqrt{4} \cdot \sqrt{10} \quad \text{(product rule for simplifying radical expressions)}
\]
\[
= 2 \sqrt{10}
\]

**EXAMPLE:** Simplify the following.

- a. \( \sqrt[3]{24} \)
- b. \( \sqrt[4]{16w^8} \)
- c. \( \sqrt[5]{54d^5} \)

**SOLUTIONS:**

- a. \( \sqrt[3]{24} = \sqrt[3]{8 \cdot 3} \quad \text{(factor 24 using perfect cube(s))} \)
  \[ = \sqrt[3]{8} \cdot \sqrt[3]{3} \quad \text{(product rule for simplifying radical expressions)} \]
  \[ = 2 \sqrt[3]{3} \]

- b. \( \sqrt[4]{16w^8} = \sqrt[4]{16} \cdot \sqrt[4]{w^8} \quad \text{(product rule for simplifying radical expressions)} \)
  \[ = \sqrt[4]{16} \cdot \sqrt[4]{(w^2)^4} \]
  \[ = 2w^2 \quad \text{(we don't need the absolute value here since } w^2 \text{ must be positive)} \]
c. \[ \sqrt{54d^5} = \sqrt{9 \cdot 6 \cdot d^4 \cdot d} \]
\[ = \sqrt{9} \cdot \sqrt{6} \cdot \sqrt{d^4} \cdot \sqrt{d} \quad \text{(product rule for simplifying radical expressions)} \]
\[ = 3d^2 \sqrt{6d} \]

Try these yourself and check your answers. Perform the indicated multiplication, and simplify completely.

a. \[ \sqrt{14} \cdot \sqrt{21} \]

b. \[ \sqrt[3]{3y^2} \cdot \sqrt[3]{9y} \]

SOLUTIONS:

a. \[ \sqrt{14} \cdot \sqrt{21} = \sqrt{14 \cdot 21} \]
\[ = \sqrt{2 \cdot 7 \cdot 3 \cdot 7} \]
\[ = \sqrt{7^2 \cdot 6} \]
\[ = 7 \sqrt{6} \]

b. \[ \sqrt[3]{3y^2} \cdot \sqrt[3]{9y} = \sqrt[3]{3y^2 \cdot 9y} \]
\[ = \sqrt[3]{27y^3} \]
\[ = 3y \]

EXAMPLE: Perform the following multiplication: \[ \sqrt[n]{x} \cdot \sqrt[n]{x} \]

SOLUTION:

The key step when the indices of the radicals are different is to write the expressions with rational exponents.

\[ \sqrt[n]{x} \cdot \sqrt[n]{x} = x^{1/n} \cdot x^{1/n} \quad \text{(write with rational exponents)} \]
\[ = x^{1/n + 1/n} \quad \text{(use a property of exponents)} \]
\[ = x^{2/n} \quad \text{(create a common denominator for the exponent)} \]
\[ = x^{7/12} \quad \text{(use another property of exponents)} \]
\[ = \sqrt[12]{x^7} \quad \text{(write final answer in radical form to agree with original expression)} \]
Try these yourself and check your answers.
Perform the indicated multiplication, and simplify completely.

a. $\sqrt{t} \cdot \sqrt[8]{t^3}$

SOLUTIONS:

a. $\sqrt{t} \cdot \sqrt[8]{t^3} = t^{1/2} \cdot t^{3/8}$ (write with rational exponents)

$= t^{1/2 + 3/8}$ (use a property of exponents)

$= t^{4/8 + 3/8}$ (create a common denominator for the exponents)

$= t^{7/8}$ (use another property of exponents)

$= \sqrt[7]{t^7}$ (write final answer in radical notation to agree with the original expression)

b. $\sqrt[3]{2p^2} \cdot \sqrt[3]{3p} = \left(2p^2\right)^{1/3} \cdot (3p)^{1/3}$ (write with rational exponents)

$= \left(2p^2\right)^{2/6} \cdot (3p)^{3/6}$ (create a common denominator for the exponents)

$= \left(\left(2p^2\right)^{2} \cdot (3p)^{3}\right)^{1/6}$

$= \left(4p^4 \cdot 27p^3\right)^{1/6}$

$= \left(108p^7\right)^{1/6}$

$= 6\sqrt[6]{108p^7}$

$= 6\sqrt[6]{108p^6p}$

$= p \cdot 6\sqrt[6]{108p}$