MTH_112: Elementary Functions - m112ps7.5 Partial Solution Set

§7.5 Simple Harmonic Motion; Damped Motion; Combining Waves

Assigned Problems: Adapted from text problems: 21, 35, 40, 53, 56

About this Worksheet

21. Graph the damped vibration curve: \( d(t) = e^{-t/\pi} \cos(2t) \) together with \( \pm E(t) \) where the "envelope" function \( E(t) = e^{-t/\pi} \) for \( 0 \leq t \leq 2\pi \). Describe the vibration in the limit as \( t \to \infty \) (i.e. as \( t \) approaches infinity).

Solution

35. Sketch a properly labelled plot of \( G(x) = \cos(4x) \cos(2x) \) together with \( \pm E(x) \) where "envelope" function is \( E(x) = \cos(2x) \) on the interval \([0, 2\pi]\).

Solution

40. An object of mass \( m = 20 \) grams attached to a coiled spring with damping factor \( b = 0.75 \) grams per second is pulled down a distance \( |a| = 15 \) centimeters from its rest position and then released. Assume that the positive direction of the motion is up and the period is \( T = 6 \) seconds under simple harmonic motion.

(a) Write an equation that relates the distance \( d \) of the object from its rest position after \( t \) seconds.

(b) Graph the equation found in part (a) for 5 oscillations using a graphing utility.

Solution

53. Tuning Fork: The end of a tuning fork moves in simple harmonic motion described by the equation \( d = a \sin(\omega t) \). If a tuning fork for the note A above middle C on an eventempered scale (\( A_4 \), the tone by which an orchestra tunes itself) has a frequency of 440 Hertz (cycles per second), find \( \omega \). If the maximum displacement of the end of the tuning fork is 0.01 millimetre, determine the equation that describes the movement of the tuning fork.

Solution

56. The Sawtooth Curve: An oscilloscope displays a sawtooth curve which can be approximated by a sum of sinusoidal curves of varying periods and amplitudes.

(a) Use a graphing utility to graph the following function, which can be used to approximate the sawtooth curve.

\[
f_1(t) = \frac{1}{\pi} \sin(\pi t) + \frac{1}{2\pi} \sin(2\pi t), \quad 0 \leq x \leq 4
\]

(b) A better approximation to the sawtooth curve is given by:

\[
f_2(t) = \frac{1}{\pi} \sin(\pi t) + \frac{1}{2\pi} \sin(2\pi t) + \frac{1}{3\pi} \sin(3\pi t)
\]

Use a graphing utility to graph this function for \( 0 \leq x \leq 4 \) and compare the result to the graph obtained in part (a).
c) A third and even better approximation to the sawtooth curve is given by

\[ f_3(t) = \frac{1}{\pi} \sin(\pi t) + \frac{1}{2\pi} \sin(2\pi t) + \frac{1}{3\pi} \sin(3\pi t) + \frac{1}{4\pi} \sin(4\pi t) \]

Use a graphing utility to graph this function for \(0 \leq x \leq 4\) and compare the result to the graphs obtained in parts (a) and (b).

(d) What do you think the next approximation to the sawtooth curve is?

Solution

END WORKSHEET