§5.6 Phase Shift; Sinusoidal Curve Fitting

Objectives

1. Graph Functions in **Standard Form**:

   \[ y = A \sin(\omega x - \phi) + B \quad \text{or} \quad y = A \sin(\omega x - \phi) + B \]

   \[ y = A \sin\left(\omega \left( x - \frac{\phi}{\omega} \right) \right) + B \quad \text{or} \quad y = A \cos\left(\omega \left( x - \frac{\phi}{\omega} \right) \right) + B \]

2. Build Sinusoidal Models from Data.

   \[ \Leftarrow \text{optional topic that we will drop} \]
§5.6 Phase Shift; Sinusoidal Curve Fitting

1. Graph Functions of the Form $y = A \sin(\omega x - \phi) + B$

One cycle of $y = A \sin(\omega x)$, $A > 0$, $\omega > 0$

\[ T = \frac{2\pi}{\omega} \]

$\omega$ is the positive real number in front of the variable
§5.6 Phase Shift; Sinusoidal Curve Fitting

1. Graph Functions of the Form \( y = A \sin(\omega x - \phi) + B \)

One cycle of \( y = A \sin(\omega x - \phi) \), \( A > 0 \), \( \omega > 0 \), \( \phi > 0 \)

For graphs of \( y = A \sin(\omega x - \phi) \) or \( y = A \cos(\omega x - \phi) \), \( \omega > 0 \)

Amplitude = \( |A| \), Period = \( T = \frac{2\pi}{\omega} \), Phase Shift = \( \phi/\omega \)

The shift is to left if \( \phi < 0 \) and to the right if \( \phi > 0 \)
§5.6 Phase Shift; Sinusoidal Curve Fitting
Steps for Graphing Sinusoidal Functions in Standard Form

Standard Form:
\[ y = A \sin(\omega x - \phi) + B \quad \text{or} \quad y = A \cos(\omega x - \phi) + B \]

**STEP 1:** Determine the amplitude \(|A|\) and period \(T = \frac{2\pi}{\omega}\).

**STEP 2:** Determine the starting point of one cycle of the graph, \(\phi/\omega\).
Determine the ending point of one cycle, \(\phi/\omega + 2\pi/\omega\). Divide the interval \([\phi/\omega, \phi/\omega + 2\pi/\omega]\) into four subintervals, each of length \(\frac{1}{4} \cdot \frac{2\pi}{\omega}\).

**STEP 3:** Use the endpoints of the subintervals to obtain five key points on the graph.

**STEP 4:** Plot the five key points with a sinusoidal graph of one cycle. Extend the graph in each direction to make it complete.

**STEP 5:** If \(B \neq 0\), apply a vertical shift.
§5.6 Phase Shift; Sinusoidal Curve Fitting

Graph Sinusoidal Functions in Standard Form

Find the amplitude, period, and phase shift of $y = -3 \cos(2x + \pi/4) + 1$ and graph the function.

Using the standard form: $y = f(x) = A \cos(\omega x - \phi) + B$

$= A \cos(\omega(x - \phi/\omega)) + B$

STEP 1: Determine the amplitude $|A|$ and period $T = 2\pi/\omega$.

$A = -3$, $T = \frac{2\pi}{2} = \pi$

STEP 2: Determine the starting point of one cycle of the graph, $\phi/\omega$.

Determine the ending point of one cycle, $\phi/\omega + 2\pi/\omega$. Divide the interval $[\phi/\omega, \phi/\omega + 2\pi/\omega]$ into four subintervals, each of length $\frac{1}{4} \cdot 2\pi/\omega$.

$\frac{\phi}{\omega} = -\frac{\pi}{8}$ $\Rightarrow$ $\left[-\frac{\pi}{8}, -\frac{\pi}{8} + \frac{1}{4}\right] = \left[-\frac{\pi}{8}, -\frac{\pi}{8} + \frac{\pi}{4}\right]$
§5.6 Phase Shift; Sinusoidal Curve Fitting

Graph Sinusoidal Functions in Standard Form \( y = -3 \cos(2x + \frac{\pi}{4}) + 1 \)

**STEP 3:** Use the endpoints of the subintervals to obtain five key points on the graph.

**STEP 4:** Plot the five key points with a sinusoidal graph of one cycle. Extend the graph in each direction to make it complete.

**STEP 5:** If \( B \neq 0 \), apply a vertical shift.

\[ y = -3 \cos \left( 2 \left( x + \frac{\pi}{8} \right) \right) + 1 \implies y \left( -\frac{\pi}{8} \right) = -3 \cos \left( 2 \left( -\frac{\pi}{8} + \frac{\pi}{8} \right) \right) + 1 = -1 \]

\[ |A| = 3 \]
§5.6 Phase Shift; Sinusoidal Curve Fitting

Graph Sinusoidal Functions in Standard Form

Find the amplitude, period, and phase shift of \( y = -3 \cos(2x + \pi/4) + 1 \) and graph the function.

Using the standard form: \( y = f(x) = A \cos(\omega x - \phi) + B \)
\[ = A \cos(\omega(x - \phi/\omega)) + B \]

Step 1: Determine the amplitude \( |A| \) and period \( T = 2\pi/\omega \)

\( A = -3 \) and \( \omega = 2 \) \( \Rightarrow \) \( T = \pi \)

Step 2: Determine the starting point of one cycle of the graph, \( \phi/\omega \).
Determine the ending point of one cycle, \( \phi/\omega + 2\pi/\omega \).
Divide the interval \( [\phi/\omega, \phi/\omega + 2\pi/\omega] \) into four subintervals, each of length \( \frac{1}{4} \cdot 2\pi/\omega \).

\( y = -3\cos(2(x + \pi/8)) + 1 \) \( \Rightarrow \) \( \phi/\omega = -\pi/8 \) and \( \phi/\omega + T = 7\pi/8 \)
so the interval of one cycle is: \([-\pi/8, 7\pi/8] \).
§5.6 Phase Shift; Sinusoidal Curve Fitting

Graph Sinusoidal Functions in Standard Form

Let $E$ be the set of subintervals end points:

$$E = \left\{ -\frac{\pi}{8}, -\frac{\pi}{8} + \frac{1}{4}T, -\frac{\pi}{8} + \frac{2}{4}T, -\frac{\pi}{8} + \frac{3}{4}T, -\frac{\pi}{8} + \frac{4}{4}T \right\}$$

$$= \left\{ -\frac{\pi}{8}, -\frac{\pi}{8} + \frac{2\pi}{8}, -\frac{\pi}{8} + \frac{4\pi}{8}, -\frac{\pi}{8} + \frac{6\pi}{8}, -\frac{\pi}{8} + \frac{8\pi}{8} \right\}$$

$$= \left\{ -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8} \right\}$$

**STEP 3:** Use the endpoints of the subintervals to obtain five key points on the graph.

Key Points = \\left\{ \left( -\frac{\pi}{8}, -3 \right), \left( \frac{\pi}{8}, 0 \right), \left( \frac{3\pi}{8}, 3 \right), \left( \frac{5\pi}{8}, 0 \right), \left( \frac{7\pi}{8}, -3 \right) \right\}
§5.6 Phase Shift; Sinusoidal Curve Fitting

Graph Sinusoidal Functions in Standard Form

**STEP 4:** Plot the five key points with a sinusoidal graph of one cycle.

**STEP 5:** If $B \neq 0$, apply a vertical shift.

\[
y = -3 \cos \left( 2x + \frac{\pi}{4} \right) + 1
\]

\[
y = -2 \quad \text{at } x = \frac{3\pi}{8}
\]

\[
y = 1 \quad \text{at } x = \frac{7\pi}{8}
\]

\[
y = 4 \quad \text{at } x = \frac{11\pi}{8}
\]
Find the amplitude, period, and phase shift of \( y = -3\cos(2x + \pi/4) + 1 \) and graph the function.

\[
y = A \cos(\omega x - \phi) + 1
\]

Subtracting from the argument of the standard form.

\[
y = -3 \cos \left( 2 \left( x + \frac{\pi}{8} \right) \right) + 1
\]

\[
A = -3
\]

\[
\omega = 2 \quad \Rightarrow \quad T = \frac{2\pi}{\omega} = \pi
\]

\[
\phi = -\frac{\pi}{4} \quad \Rightarrow \quad \frac{\phi}{\omega} = -\frac{\pi}{8}
\]
The specified point in the Figure is \((t, y) = (2\pi, 2)\). **Present the analysis** to find a function \(y = f(t)\) as a sine function AND a cosine function whose graph would appear as shown in the Figure.
\[ f(t) = A \cos \left( \omega (t - \frac{\Phi}{\omega}) \right) + B \]

\[ = \sqrt{2} \cos \left( \omega (t - 0) \right) + 4 \]

\[ = 2 \cos \left( \frac{1}{2} t \right) + 4 \]

\[ f(2\pi) = 2 \cos \left( \frac{1}{2} \cdot 2\pi \right) + 4 \quad \text{Sanity check!} \]

\[ T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2} \]
Tech check

\[ y = 2 \cdot \cos \left( \frac{1}{2} x \right) + 4 \]

\[ y = 2 \cdot \sin \left( \frac{1}{2} (x + \pi) \right) \]

\[ y = 4 \]

\[ y = 6 \]

\[ y = 2 \]
$S(t) = A \sin(\omega (t - \frac{\phi}{\omega}))+M$

$= 2 \sin(\frac{1}{2}(t-\frac{\phi}{\omega}))+4$

$= 2 \sin(\frac{1}{2}(t+\pi))+4$

Sanity check $S(2\pi) = 2 \sin\left(\frac{1}{2}(2\pi+\pi)\right)+4$

$= 2 \sin\left(\frac{5\pi}{2}\right)+4$

$= -2+4$

$= 2$
§5.6 Phase Shift; Sinusoidal Curve Fitting

2. Build Sinusoidal Models from Data

Assume the data can be fit to:

\[ y = A \cos(\omega x - \phi) + B \]
## §5.6 Phase Shift; Sinusoidal Curve Fitting

### 2. Build Sinusoidal Models from Data

**STEP 1: Determine \( |A| \)**

\[
|A| = \frac{y_{\max} - y_{\min}}{2}
\]

\[
|A| = \frac{73.5 - 29.7}{2} = 21.9
\]

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<tr>
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§5.6 Phase Shift; Sinusoidal Curve Fitting

2. Build Sinusoidal Models from Data

### STEP 2: Determine $B$

$$B = \frac{y_{\text{max}} + y_{\text{min}}}{2}$$

$$B = \frac{73.5 + 29.7}{2} = 51.6$$

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§5.6 Phase Shift; Sinusoidal Curve Fitting

2. Build Sinusoidal Models from Data

**STEP 3: Determine $\omega = \frac{2\pi}{T}$**

$$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$y = -21.9 \cos\left(\frac{\pi}{6}x\right) + 51.6$$

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§5.6 Phase Shift; Sinusoidal Curve Fitting

2. Build Sinusoidal Models from Data

**STEP 4:** Determine the horizontal shift of the function by using the period of the data. Divide the period into four subintervals of equal length. Determine the $x$-coordinate for the maximum of the sinusoid and the $x$-coordinate for the maximum of the data. Use this information to determine the value of the phase shift, $\phi/\omega$. 

$$y = -21.9 \cos \left( \frac{\pi}{6} (x - 1) \right) + 51.6$$
**Steps for Fitting Data to a Sine Function** $y = A \sin(\omega x - \phi) + B$

**STEP 1:** Determine $A$, the amplitude of the function.

\[
\text{Amplitude} = \frac{\text{largest data value} - \text{smallest data value}}{2}
\]

**STEP 2:** Determine $B$, the vertical shift of the function.

\[
\text{Vertical shift} = \frac{\text{largest data value} + \text{smallest data value}}{2}
\]

**STEP 3:** Determine $\omega$. Since the period $T$, the time it takes for the data to repeat, is $T = \frac{2\pi}{\omega}$, we have

\[
\omega = \frac{2\pi}{T}
\]

**STEP 4:** Determine the horizontal shift of the function by using the period of the data. Divide the period into four subintervals of equal length. Determine the $x$-coordinate for the maximum of the sine function and the $x$-coordinate for the maximum value of the data. Use this information to determine the value of the phase shift, $\frac{\phi}{\omega}$. 

These may be useful.
### §5.6 Phase Shift; Sinusoidal Curve Fitting

**Fit a Sinusoidal Model with a Graphing Utility**

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§5.6 Phase Shift; Sinusoidal Curve Fitting
Fit a Sinusoidal Model with a Graphing Utility

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[Graphing Utility Image]
§5.6 Phase Shift; Sinusoidal Curve Fitting

Fit a Sinusoidal Model with a Graphing Utility
§5.6 Phase Shift; Sinusoidal Curve Fitting
Fit a Sinusoidal Model with a Graphing Utility

Stat Calculation
Sinusoidal Reg
y = a \cdot \sin(b \cdot x + c) + d

a = 21.145556
b = 0.5495633
c = -2.350732
d = 51.196763
MSe = 2.153219

Zoom Analysis Calc

list1 list2 list3
1 1 29.7
2 2 33.4
3 3 39
4 4 48.2
5 5 57.2

Rad Auto Decimal

[ 13] =

82

0.1

20

Kidoguchi, Kenneth
§5.6 Phase Shift; Sinusoidal Curve Fitting

Fit a Sinusoidal Model with a Graphing Utility

\[ y_{eyeball} = -21.9 \cos \left( \frac{\pi}{6} (x - 1) \right) + 51.6 \]

\[ y_{Casio} = 21.145556 \sin(0.5495633 x - 2.350732) + 51.196763 \]