Limit Law Sheet for MTH 251 Lab 2

• Page 1 - Real Number Limit Laws from section 2.3 of the Stewart Text
• Page 2 - Infinite Limit Law and Limit at Infinity Laws from Section 2.5 of the Stewart Text

Limit Laws 2.3.1 - 2.3.11 are valid if and only if:

• $p$ and $C$ are real numbers.
• Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist; i.e. each limit is a real number.

Limit Law 2.3.1:  \[
\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)
\]

Limit Law 2.3.2:  \[
\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)
\]

Limit Law 2.3.3:  \[
\lim_{x \to a} (C \cdot f(x)) = C \cdot \lim_{x \to a} f(x)
\]

Limit Law 2.3.4:  \[
\lim_{x \to a} (f(x) \cdot g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)
\]

Limit Law 2.3.5:  \[
\lim_{x \to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}
\]

\begin{center}
Law 2.3.5 is valid if and only if $\lim_{x \to a} g(x) \neq 0$.
\end{center}

Limit Law 2.3.6:  \[
\lim_{x \to a} \left[ (f(x))^p \right] = \left( \lim_{x \to a} f(x) \right)^p
\]

Limit Law 2.3.7:  \[
\lim_{x \to a} C = C
\]

Limit Law 2.3.8:  \[
\lim_{x \to a} x = a
\]
Infinity Limit Laws

Limit Law 2.3.9: \( \lim_{x \to a} x^n = a^n \)

Law 2.3.9 is valid if and only if \( n \) is a positive integer.

Limit Law 2.3.10: \( \lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a} \)

If \( n \) is even, we assume that \( a > 0 \).

Limit Law 2.3.11: \( \lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{f(a)} \)

Where \( n \) is a positive integer. If \( n \) is even, we assume that \( f(x) > 0 \).

In each of Limit Laws 2.3.1 - 2.3.11 the letter \( a \) can represent a real number, the symbol \( \infty \), or the symbols \( -\infty \).

Limit Law 2.5.3: \( \lim_{x \to a^+} \ln x = -\infty \)

Limit Law 2.5.6a: \( \lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2} \)

Limit Law 2.5.6b: \( \lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2} \)

Limit Law 2.5.7

If \( p \) is a positive real number and \( C \) is any real number, then \( \lim_{x \to \infty} \frac{C}{x^p} = \lim_{x \to -\infty} \frac{C}{x^p} = 0 \)

Limit Law 2.5.8: \( \lim_{x \to -\infty} e^x = 0 \)

"Vertical Asymptote/Infinite Limit" Law

If \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) both exist (as real numbers) and \( \lim_{x \to a} f(x) \neq 0 \) and \( \lim_{x \to a} g(x) = 0 \),

then \( \lim_{x \to a} \left( \frac{f(x)}{g(x)} \right) \) does not exist!
NOTE

In this situation, we sometimes write that the limit "equals infinity" or "equals negative infinity". For example we write \( \lim_{x \to 0^+} \frac{1}{x} = \infty \). This expression is simply short hand for saying \( \lim_{x \to 0^+} \frac{1}{x} \) does not exist because as the value of \( x \) gets closer and closer to zero from the right, the value of \( \frac{1}{x} \) becomes larger and larger positive numbers with no limit on how large the value of \( \frac{1}{x} \) gets.