MTH 251 Lab 4 Write-up Example

Activity 3.z

I decided to take out a five year loan to buy a car. \( P = f(r) \) is the payment I will have to make (in dollars/month) if the annual interest rate is \( r \).

For example, \( f(0.10) = 250 \) means that if the annual interest rate on the loan is 10\%, then the loan payment will be 250 $/month.

\[ f'(230) = 0.0875 \] means that if I want the payment to be only 230 $/month, then I need the annual interest rate to be 8.75\%.

The units on \( f'(0.10) = 15.7 \) are \( \frac{\text{$/month}}{\text{percentage point}} \). The meaning of this derivative value is that at an annual interest rate of 10\%, the payment is increasing at a rate of 15.7 $/month per percentage point. So if the payment is 250 $/month when the annual interest rate is 10\%, then the payment is about 265.70 $/month if the annual interest rate is 11\%. The payment at an annual interest rate of 11\% is only an estimate because the rate of change in the function \( f(r) \) is not constant - i.e. \( f(r) \) is not a linear function.

Activity 2.x

Figure 1 shows a plot of the function \( f(x) = 2x^3 + 3x^2 - 36x + 4 \).

At any given value of \( x \), the value of \( f'(x) \) is the slope of the tangent line to \( f(x) \) at that value of \( x \). Clearly the tangent line to \( f(x) \) is horizontal when \( x = 3 \) and when \( x = 2 \), so \( f'(x) = 0 \) at \( x = 3 \) and \( x = 2 \). The tangent line to \( f(x) \) has negative slope for \( 3 < x < 2 \) and the tangent line to \( f(x) \) has positive slope for \( x < 3 \) and for \( x > 2 \); consequently \( f'(x) < 0 \) for \( 3 < x < 2 \) and \( f'(x) > 0 \) for \( x < 3 \) and for \( x > 2 \).

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The formula for the function $f'(x)$ is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^3 + 3(x+h)^2 - 36(x+h) + 4(2x^3 + 3x^2 - 36x + 4)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^3 + 3x^2 h + 3xh^2 + h^3) + 3(x^2 + 2xh + h^2) - 36(x+h) + 4(2x^3 + 3x^2 + 36x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{2x^3 + 6x^2 h + 6xh^2 + 2h^3 + 3x^2 + 6xh + 3h^2 - 36x + 36h + 4(2x^3 + 3x^2 + 36x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{6x^2 h + 6xh^2 + 2h^3 + 6xh + 3h^2 - 36h}{h}$$

$$= \lim_{h \to 0} \left(6x^2 + 6x + 2h^2 + 6x + 3h^2 - 36\right)$$

$$= 6x^2 + 6x - 36$$

Figure 2 shows that $f'(x) = 6x^2 + 6x - 36$ is indeed positive when $f(x)$ is increasing (i.e. when $x < -3$ and when $x > 2$) and $f'(x) = 6x^2 + 6x - 36$ is indeed negative when $f(x)$ is decreasing (i.e. when $3 < x < 2$). 😊

Figure 2: $f(x) = 2x^3 + 3x^2 - 36x + 4$ is the thin curve $f'(x) = 6x^2 + 6x - 36$ is the thick curve