Sample Problems # 2

1. Implicitly differentiate the following and find \( \frac{dy}{dx} \).

\[
\sin(y) = 4x^2 + y
\]
Below is the graph of the function \( f(x) \). If \( F \) is an antiderivative of \( f \) and \( f' \) is the derivative of \( f \), answer each of the following questions. Note that “nowhere”, “on no interval”, or “there is no way of knowing” are possible responses to the questions. No explanations are required. Be careful to distinguish between \( F \), \( f \) and \( f' \).

Where is \( f \) nondifferentiable?

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Where is \( f'(x) = 0 \)?

On what interval(s) is \( f' \) decreasing?

On what interval(s) is \( F \) increasing?

On what interval(s) is \( f \) concave down?

On what interval(s) is \( F \) concave down?

On what interval(s) is \( f \) negative?

Note: \( \lim_{x \to -\infty} f(x) = 1/2 \)
3. Suppose that \( f(x) = \ln(g(x)) \). Suppose further that \( g(-3) = 2 \), \( g\left(\frac{1}{2}\right) = 8 \), \( g'(-3) = \frac{2}{3} \), and \( g'\left(\frac{1}{2}\right) = 7 \). Find \( f'(-3) \).
4. Use the process of **logarithmic differentiation** to find the first derivative of the function \( y = x^{\ln(x)} \). Completely simplify the result.
5. This is the graph of $y = f(x)$.

a. Draw the graph of $y = f'(x)$ below.
6. Sketch onto the grid a function that satisfies each of the following properties.

- \( f'(0) = f'(4) = 0 \)
- \( f'(x) > 0 \) on \((-\infty, 0)\)
- \( f'(x) < 0 \) on \((0, 4)\) and \((4, \infty)\)
- \( f''(x) > 0 \) on \((2, 4)\)
- \( f''(x) < 0 \) on \((-\infty, 2)\) and \((4, \infty)\)
7. Use the trigonometric facts and the quotient rule to derive the formula for the
derivative of secant. You must get the same formula for \( \frac{d}{dx} (\sec(x)) \) as the one in
the textbook.

\[
f(x) = \sec(x)
\]
8. The water level (ft) inside a nuclear reactor is modeled by

\[ f(t) = 3 \cos(t^2) + 2t + 15 \quad \text{for } 0 \leq t \leq 20 \quad \text{(in mins)} \]

a. Find \( f'(t) \).

b. What is the instantaneous rate of change of the water level at \( t = 0.5 \)? (Give Unit) (Give 4 decimal places.)

c. Use your calculator to state the first time, call it \( t_1 \), after \( t = 0 \) such that \( f'(t) = 0 \). (Give Unit) (Give 4 decimal places.)