4.90 In 2000, the Internal Revenue Service received 129,075,000 individual tax returns. Of these 10,855,000 reported an adjusted gross income of at least $100,000, and 240,000 reported at least $1 million. If you know that a randomly chosen return shows an income of $100,000 or more, what is the conditional probability that the income is at least $1 million?

Step 1 – Write down probabilities using function notation.

\[ P(> \$100 \text{ K}) = \frac{10,855,000}{129,075,000} = 0.084098 \]

\[ P(> \$1 \text{ million}) = \frac{240,000}{129,075,000} = 0.001859 \]

\[ P(> \$1 \text{ million} \mid > \$100 \text{ K}) = \frac{0.001859}{0.084098} = 0.022105 \]

Which is the same as if we used the actual counts.

\[ P(> \$1 \text{ million} \mid > \$100 \text{ K}) = \frac{240,000}{10,855,000} \]
4.96 Here are the counts (in thousands) of earned degrees in the U.S. in the 2005-2006 academic year, classified by level and the by the sex of the degree recipient.

<table>
<thead>
<tr>
<th></th>
<th>Bachelor’s</th>
<th>Master’s</th>
<th>Professional</th>
<th>Doctorate</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>784</td>
<td>276</td>
<td>39</td>
<td>20</td>
<td>1119</td>
</tr>
<tr>
<td>Male</td>
<td>559</td>
<td>197</td>
<td>44</td>
<td>25</td>
<td>825</td>
</tr>
<tr>
<td>Total</td>
<td>1343</td>
<td>473</td>
<td>83</td>
<td>45</td>
<td>1944</td>
</tr>
</tbody>
</table>

a) If you choose a degree recipient at random, what is the probability that the person you choose is a woman?

\[
P(\text{woman}) = \frac{1119}{1944} \approx 0.5756
\]

b) What is the conditional probability that you choose a woman, given that the person chosen is received a professional degree?

\[
P(\text{woman} \mid \text{professional degree}) = \frac{39}{83} \approx 0.4699
\]

c) Are the events “choose a woman” and “choose a professional degree recipient” independent? How do you know?

Since \( P(\text{woman} \mid \text{professional degree}) \neq P(\text{woman}) \), the two events cannot be independent.