1. **Heights of women.** The heights of women aged 20 to 29 are approximately Normal with mean 64 inches and standard deviation 2.7 inches. Men the same age have mean height 69.3 inches with standard deviation 2.8 inches, also normally distributed. Let the random variable \( Y \) determine the height of a woman in inches.

a. What is the probability that a randomly chosen woman is either shorter than 60 inches, or taller than 69 inches?

\[
P(Y < 60 \text{ OR } Y > 69) = P(Y < 60) + P(Y > 69) \quad \text{The events are disjoints thus.}
\]

\[
= P \left( Z < \frac{60 - 64}{2.7} \right) + P \left( Z > \frac{69 - 64}{2.7} \right)
\]

\[
= P(Z < -1.48) + P(Z > 1.85)
\]

\[
= 0.0694 + 0.0322
\]

\[
= 0.1016
\]

b. What is the probability that a randomly chosen woman is over 64 inches tall and less than 70 inches tall?

\[
P(Y > 64 \text{ AND } Y < 70) = P(64 < Y < 70) \quad \text{Restate the question.}
\]

\[
= P(Y < 70) - P(Y < 64) \quad \text{Represents what I will have to do to answer question}
\]

\[
= P \left( Z < \frac{70 - 64}{2.7} \right) - 0.5
\]

\[
= P(Z < 2.22) - 0.5
\]

\[
= 0.9868 - 0.5
\]

\[
= 0.4868
\]

c. What is the probability that a randomly chosen woman is 65 inches or taller and less than 60 inches?

Let the random variable \( W \) be the height of a randomly chosen woman. \( P(W > 65 \text{ AND } W < 60) = 0 \), since one person cannot satisfy both criteria simultaneously.

d. Are the events mentioned in “c” disjoint? **Yes, since one person cannot satisfy both criteria simultaneously.**
e. Let the random variable $X$ measure the height of a male in inches. Calculate $P(X > 75.4 \text{ OR } X < 69.3)$

- Men the same age have mean height 69.3 inches with standard deviation 2.8 inches.

$$P(X > 75.4 \text{ OR } X < 69.3) = P(X > 75.4) + P(X < 69.3) \text{ Events are disjoint so we can separate.}$$

$$= P\left(Z > \frac{75.4 - 69.3}{2.8}\right) + P\left(Z < \frac{69.3 - 69.3}{2.8}\right)$$

$$= P(Z > 2.18) + 0.5$$

$$= 0.0146 + 0.5$$

$$= 0.5146$$

Note: $P(Z < 2.18) = 0.9854$, so $P(Z > 2.18) = 1 - 0.9854 = .0146$

2. I set the random number generator in Excel or CrunchIt! to generate random numbers from a uniform distribution.

The computer can generate any real numbers between 5 and 30. The random variable $X$ represents all the random numbers generated by the computer.

a. What is the sample space of the random variable $X$?

**All the real numbers between 5 and 30 including 5 and 30.**

b. What is the probability that a random number generated by the computer program is either less than 12 or greater than 22?

$$P(X < 12 \text{ or } X > 22) = P(X < 12) + P(X > 22) \text{ The two events are disjoint}$$

$$= \frac{1}{25}(12 - 5) + \frac{1}{25}(30 - 22)$$

$$= 0.28 + 0.32$$

$$= 0.6$$

c. What is the probability that a random number generated by the computer is either greater than 15 or less than 20?

$$P(X > 15 \text{ or } X < 20) = 1$$

Since, as you can see from the picture $X > 15 \text{ or } X < 20$ encompasses the entire sample space.

$$P(X > 15 \text{ or } X < 20) = P(5 < X < 30)$$

$$= 1$$
d. What is the probability that a random number generated by the computer is both less than 15 and more than 29?

\[ P(X < 15 \text{ and } X > 29) = 0 \]

One observation cannot satisfy both events simultaneously.

\[
\begin{align*}
\text{Domain of } X & = (29, 30) \\
\text{Height of } X & = \frac{1}{25} \\
\text{Area of } X & = \frac{1}{25} \\
\text{Mean } \mu & = 17.5 \\
\text{Standard Deviation } \sigma & = 7.22
\end{align*}
\]

e. Are the events mentioned in part “d” disjoint? Yes, one number generated at random cannot satisfy both events simultaneously.

f. \[ P(X < 24 \text{ AND } X > 20) = P(20 < X < 24) \]

Restate the question

\[ \frac{1}{25} (24 - 20) \]

\[ = 0.16 \]

3. A coin is tossed 12 times into the air. The random variable \( X \) counts the number of times that a coin lands heads. Write down the sample space of the random variable \( X \).

\[ \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \]

4. A MTH 243 class has 35 students. Out of those 35 students 8, have taken the course the previous term but did not pass. The instructor for the class will sample 6 students at random and look at their transcripts. Let the random variable \( Y \) count the number of students out of the sample of six that did not pass the previous term. Write down the sample space of the random variable \( Y \).

\[ \{0, 1, 2, 3, 4, 5, 6\} \]
5. A test is created to test if a person has been infected with HIV. The test is an over the counter exam, performed by the individual. If a person is infected it will detect this 80% of the time. Suppose that five people with HIV are tested using this exam. Let the random variable $H$ count how many out of the five are correctly identified as having HIV. Below are the values of the random variable $H$.

<table>
<thead>
<tr>
<th>$H$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>0.0003</td>
<td>0.0064</td>
<td>0.0512</td>
<td>0.2048</td>
<td>0.4096</td>
<td></td>
</tr>
</tbody>
</table>

a. What is the probability that all of the five are correctly identified?

$$1 - (0.0003 + 0.0064 + 0.0512 + 0.2048 + 0.4096) = 0.3277$$

b. What is the sample space of the random variable $H$?  \{0, 1, 2, 3, 4, 5\}

c. What is the probability that out of a sample of five either 1 or 4 people have been detected as having an HIV infection?

$$P(H = 1 \ or \ H = 4) = P(H = 1) + P(H = 4) \ Events \ are \ disjoint.$$  

$$= 0.0064 + 0.4096$$

$$= 0.416$$

d. Out of a single sample of five HIV infected people, calculate $P(H = 0 \ AND \ H = 3)$?

$$P(H = 0 \ AND \ H = 3) = 0 \ since \ one \ observation \ (recall \ one \ observation \ consist \ of \ the \ result \ of \ testing \ five \ individuals) \ cannot \ satisfy \ both \ events \ simultaneously.$$  

e. Calculate $P(H = 2 \ or \ H = 4 \ or \ H = 5) = P(H = 2) + P(H = 4) + P(H = 5) \ Events \ are \ disjoint.$

$$= 0.0512 + 0.4096 + 0.3277$$

$$= 0.7885$$