1.99 Test scores. Many states have programs for assessing the skills of students in various grades. The Indiana Statewide Testing for Educational Progress (ISTEP) is one such program. In a recent year 76,531 tenth-grade Indiana students took the English/language arts exam. The mean score was 572 and the standard deviation was 51. Assuming that these scores are approximately Normally distributed, N(572, 51), use the 68-95-99.7 rule to give a range of scores that include 95% of these students.

Let the random variable Y denote scores on the ISTEP. We want to find \( P(a < Y < b) \approx 0.95 \)

Using the 68-95-99.7 rule, \( a = 572 - 2(51) = 470 \), and \( b = 570 + 2(51) = 674 \). Thus, the required range of scores is (470, 674).

1.101 Find the z-score. Consider the ISTEP scores (see Exercise 1.99), which we can assume are approximately Normal, N(572, 51). Give the z-score for a student who received a score of 600.

I will use the formula \( Z = \frac{X - \mu}{\sigma} \) to find the corresponding value.

\[
Z = \frac{600 - 572}{51} \approx 0.5490.
\]
This value tells me that I am a little more than half a standard deviation to the right of the mean 572.
1.103 Find the proportion. Consider the ISTEP scores (see Exercise 1.99), which we can assume are approximately Normal, N(572, 51). Find the proportion of students who have scores less than 600. Find the proportion of students who have scores greater than or equal to 600. Sketch the relationship between these two calculations using pictures of Normal curves similar to the ones given in Example 1.27.

\[ P(Y > 600) = P\left( Z > \frac{600 - 572}{51} \right) \]

\[ = P(Z > 0.5490) \]

\[ = 0.2915 \]

I entered 0.5490, made sure the inequality symbol read \( => \), that mean is set to 0 and Std. dev. to 1, and finally hit compute.

On Excel I would have to type: \( =1 – \text{normsdist}(0.5490) \); notice that Excel can only calculate in one direction: \(<\). I want \( > \) I need to get a bit clever and use the appropriate facts about density curves.

\[ P(Y < 600) = P\left( Z < \frac{600 - 572}{51} \right) \]

\[ = P(Z < 0.5490) \]

\[ = 0.7085 \]

I entered 0.5490, made sure the inequality symbol read \( <= \), that mean is set to 0 and Std. dev. to 1, and finally hit compute.

On Excel I would have to type: \( =\text{normsdist}(0.5490) \).

The pictures suggest that the sum of my two answers has to be one; 0.2915 + 0.7085 = 1. This makes sense since this is a density curve, area is always 1, and the two intervals encompass the entire normal distribution.
**1.105 What score is needed to be in the top 5%?** Consider the ISTEP scores, which are approximately Normal, N(572, 51). How high a score is needed to be in the top 5% of students who take this exam?

What is the value of y such that P(Y > y) = 0.05? What I can do, is find that value’s z-score; with software it is possible not to do the following steps, but, it would be a great mistake. The z-score is as important of a concept as the value we are trying to find and the connection between the two is critical; this is not just an academic exercise. It is common to use this type of communication when using statistics.

I will use software to find the associated z-score $P(Z > z) = 0.05; z \approx 1.645$.

Now I will use the formula $x = Z\sigma + \mu$ (derived by solving $Z = \frac{x - \mu}{\sigma}$ for $x$), to find the missing value.

\[
y = 1.645(51) + 572 = 655.9; \text{ that is, } P(Y > 655.9) \approx 0.05. \text{ A score of 655.9 will put you in the top 5%}.
\]

**1.108 A uniform distribution.** If you ask a computer to generate "random numbers between 0 and 1, you will get observations from a **uniform distribution**. Figure 1.37 graphs the distribution density curve for a uniform distribution. Use areas under this density curve to answer the following questions. **Define the random variable X to be the value that is generated by the computer.**

![Uniform Distribution](image)

**Figure 1.37 The density curve of a uniform distribution, for exercise 1.108.**

(a) Why is the total area under this curve equal to 1? **Since the figure is a defined as a density curve, then by definition it has a total area of 1 square unit.**

(b) What proportion of the observations lie below 0.35?

**Let the variable X denote the values generated by the random number generator. To answer this question we need only to find the area below the curve corresponding to X < 0.35.**
Keep in mind that because the author chose a uniform distribution with endpoints 0 and 1, it is easy to see what the proportion should be without much thought. Make sure you learn the real lesson here, that in order to calculate proportions with density curves, the area underneath the curve is directly related to the corresponding proportion.

(c) What proportion of the observations lie between 0.35 and 0.65?

\[
P(0.35 < X < 0.65) = 1(0.65 - 0.35) = 0.4
\]

1.115 **Horse pregnancies are longer.** Bigger animals tend to carry their young longer before birth. The length of horse pregnancies from conception to birth varies according to a roughly normal distribution with mean 336 days and standard deviation 3 days. Use the 68-95-99.7 rule to answer the following questions.

(a) Almost all (99.7%) horse pregnancies fall in what range of lengths?

Since the instructions said to use the 68-95-99.7 rule, then I must come to the conclusion that the 99.7% range the question asks for is the middle 99.7%.

Thus the range is \( \pm 3\sigma \) from \( \mu \).

\[
336 - 3(3) = 227
\]

\[
336 + 3(3) = 345 \quad \text{The range is from 227 days to 345 days.}
\]

(b) What percent of horse pregnancies are longer than 339 days?

Since, 339 days is one standard deviation to the right of the mean 336, we can use the 68-95-99.7 rule along with the fact that the total area under the curve is 1 square unit (100%) to get the answer.

Let \( X \) equal the length of a horse pregnancy in days.

\[
P(X > 339 \text{ days}) = \frac{1 - 0.68}{2} = 0.16.
\]
1.117 Heights of women. The heights of women aged 20 to 29 are approximately Normal with mean 64 inches and standard deviation 2.7 inches. Men the same age have mean height 69.3 inches with standard deviation 2.8 inches. What are the z-scores for a woman 6 feet tall and a man 6 feet tall? What information do the z-scores give that the actual heights do not?

I will use the z-score formula \( Z = \frac{x - \mu}{\sigma} \).

For a woman that is 6 feet tall, 72 inches, her corresponding z-score is given by: 
\[
z = \frac{72 - 64}{2.7} \approx 2.96
\]

For a man that is 6 feet tall, 72 inches, his corresponding z-score is given by: 
\[
z = \frac{72 - 69.3}{2.8} \approx 0.9642
\]

The two z-scores show that a woman that is 6 feet tall is very tall compared to other women. If the distribution is normal then we know that being almost 3 standard deviations to the right, puts this woman in a height that is rare among women. Compared to a man with the same height we see that the man would be above average since he is almost one standard deviation to the right of the mean, but, that height is not among the tallest for men, nor is it too unusual for men (assuming a normal distribution). Thus, relatively speaking, the woman is much taller than the man with respect to their gender.

1.123 Find more values of z. The variable Z has a standard normal distribution.

What is the purpose of this problem? To continue your flexibility with the use of either tables, or a computer system to generate values associated with the normal distribution, and ALSO to make sure you visually understand the meaning of each value you generate and how it relates to the normal distribution graph. LASTLY, to make sure you are using the z-score concept; very important.

a) Find the number z that has cumulative proportion 0.85.

The word cumulative means to add up, combine, or aggregate. When you see the word cumulative in statistics we are “adding” from left to right; thus, if we had the values \{1, 4, 5, 7\} The cumulative sum as I go from left to right is \{1, 5, 10, 17\}. Visually and using symbols the above question appears as

\[
P(Z < z) = 0.85, \text{ what is } z?
\]
Using CrunchIt! the result is ≈ 1.0364.

On Excel there is no visual, but the command is =normsinv(0.85) this will produce the value of 1.0364.

b) Find the number z such that the event Z > z has proportion 0.40.

The question is what value of z corresponds to a proportion equal to 0.4; P(Z > z) = 0.4?

The value of z = 0.2533.

On Excel I would have to type: =normsinv(0.6); notice that Excel can only calculate in one direction: <. I want > I need to get a bit clever and use the appropriate facts about density curves.
1.131 **Find the ACT percentile.** Reports on a student’s ACT or SAT usually give the percentile as well as the actual score. The percentile is just the cumulative proportion stated as a percent: the percent of all scores that were lower than this one. Jacob scores 17 on the ACT. What is his percentile?

Let the random variable $X$ denote the score on the ACT.

For the ACT test the mean is $\mu = 20.8$, and the standard deviation is $\sigma = 4.8$ and is roughly normal.

Jacob scores a 17, his percentile is given by

$$P(X < 17) = P\left(Z < \frac{17 - 20.8}{4.8}\right) = P(Z < -0.7917) = 0.2143$$

To get the value of 0.2143, I can use CrunchIt! or Excel.

On Excel I would type the command, $=\text{normsdist}(-0.7917)$. I entered -0.7917, made sure the inequality symbol read $<=$, that mean is set to 0 and Std. dev. to 1, and finally hit compute.