Physics 202 Homework 8
May 22, 2013

1. A beam of sunlight encounters a plate of crown glass at a 45.00° angle of incidence. Using the data in Figure 1, find the angle between the violet ray and the red ray in the glass.

<table>
<thead>
<tr>
<th>Color</th>
<th>Vacuum Wavelength (nm)</th>
<th>Index of Refraction, n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>660</td>
<td>1.520</td>
</tr>
<tr>
<td>Orange</td>
<td>610</td>
<td>1.522</td>
</tr>
<tr>
<td>Yellow</td>
<td>580</td>
<td>1.523</td>
</tr>
<tr>
<td>Green</td>
<td>550</td>
<td>1.526</td>
</tr>
<tr>
<td>Blue</td>
<td>470</td>
<td>1.531</td>
</tr>
<tr>
<td>Violet</td>
<td>410</td>
<td>1.538</td>
</tr>
</tbody>
</table>

*Approximate

Figure 1: Problem 26.41

Solution

For the red ray, \( n = 1.520 \). According to Snell’s law, the angle of refraction inside the glass is

\[
(1) \sin(45.00^\circ) = (1.520) \sin \theta \implies \theta = 27.723^\circ
\]

And for the violet ray, \( n = 1.538 \). So,

\[
(1) \sin(45.00^\circ) = (1.538) \sin \theta \implies \theta = 27.371^\circ
\]

The difference is

\[27.723^\circ - 27.371^\circ = 0.352^\circ\]

2. You are trying to photograph a bird sitting on a tree branch, but a tall hedge is blocking your view. However, as Figure 2 shows, a plane mirror reflects light from the bird into your camera. For what distance must you set the focus of the camera lens in order to snap a sharp picture of the bird’s image?

Figure 2: Problem 25.3

Solution
The image of the bird is located into the mirror at the same distance the object is from the mirror, in this case 2.1 meters. This means the horizontal distance from the camera to the image is the 3.7 meters from the camera to the mirror and an additional 2.1 meters to the image, or 5.8 meters total. The vertical distance is still 4.3 meters. With the image on the other side, a right triangle is formed. By the Pythagorean theorem, the distance is

\[ d = \sqrt{(5.8)^2 + (4.3)^2} = 7.2201 \]

3. A small statue has a height of 3.5 cm and is placed in front of a concave mirror. The image of the statue is inverted, 1.5 cm tall, and is located 13 cm in front of the mirror. Find the focal length of the mirror.

Solution

By definition, the magnification is \( m = h_i/h_o \). In this case, since the image is inverted, the image height is negative. Thus,

\[ m = \frac{-1.5}{3.5} = -0.42857 \]

The magnification tells us something about the image and object distances according to \( m = -d_i/d_o \). Thus,

\[ (-0.42857) = -\frac{d_i}{d_o} \]

Since we know the image distance, we can calculate the object distance:

\[ (-0.42857) = -\frac{13}{d_o} \implies d_o = 30.333 \]

We can now use the lens equation \( (1/d_o + 1/d_i = 1/f) \) to calculate the focal length:

\[ \frac{1}{30.333} + \frac{1}{13} = \frac{1}{f} \implies f = 9.1000 \]

4. The outside mirror on the passenger side of a car is convex and has a focal length of -7.0 meters. Relative to this mirror, a truck traveling in the rear has an object distance of 11 meters. Find (a) the image distance of the truck and (b) the magnification of the mirror.

Solution

(a) The lens equation is

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]

Thus,

\[ \frac{1}{11} + \frac{1}{d_i} = \frac{1}{-7} \implies d_i = -4.2778 \]

(b) The magnification can be derived from

\[ m = -\frac{d_i}{d_o} \]

thus,

\[ m = -\frac{-4.2778}{11} = 0.38889 \]

5. A spherical mirror is polished on both sides. When the convex side is used as a mirror, the magnification is 0.25. What is the magnification when the concave side is used as a mirror, the object remaining the same distance from the mirror?
Solution

The magnification equation states $m = -d_i/d_o$, so

$$d_i = - (0.25) d_o$$

The lens equation states $1/d_o + 1/d_i = 1/f$, so

$$\frac{1}{d_o} + \frac{1}{-(0.25)d_o} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{d_o} + \frac{-4}{d_o} = \frac{1}{f}$$

$$\Rightarrow \frac{-3}{d_o} = \frac{1}{f}$$

$$\Rightarrow f = -d_o/3$$

Now if we flip the mirror the new focal length is simply the original with the sign flipped. Thus,

$$f = d_o/3$$

We can plug this into another lens equation to get

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{3}{d_o}$$

$$\Rightarrow \frac{1}{d_i} = \frac{2}{d_o}$$

$$\Rightarrow d_i = d_o/2$$

Therefore the magnification must be

$$m = \frac{-d_i}{d_o} = \frac{-d_o/2}{d_o} = -0.5000$$

6. Figure 3 shows a top view of a square room. One wall is missing, and the wall on the right is a mirror. From point $P$ in the center of the open side, a laser is pointed at the mirrored wall. At what angle of incidence must the light strike the right-hand wall so that, after being reflected, the light hits the left corner of the back wall?

![Figure 3: Problem 25.47](image)

Solution

Let the length of each side of the room be $L$. The trick is to find the image of this corner in beyond the mirror. It is simply on the other side one wall-length inside the mirror. The laser must point at this image. The laser beam will run along the hypotenuse of a right triangle with the image of the far wall as the opposite side (with length $L$) and the adjacent side is from the laser to the image (therefore a length $\frac{1}{2}L + L$). The angle we are looking for is obtained by the tangent function:

$$\tan \theta = \frac{L}{(1.5)L} = 0.66667 \Rightarrow \theta = 33.690^\circ$$
7. A spotlight on a boat is 2.5 meters above the water, and the light strikes the water at a point that is 8.0 meters horizontally displaced from the spotlight (see Figure 4). The depth of the water is 4.0 meters. Determine the distance $d$, which locates the point where the light strikes the bottom.

![Figure 4: Problem 26.12](image)

**Solution**

The key here is to apply Snell’s law at the point of refraction. The incident angle is the complement to the acute angle formed by the light ray and the surface of the water. The tangent of this acute angle is related to the distances in the diagram:

$$\tan \theta = \frac{2.5}{8.0} \implies \theta = 17.354^\circ$$

So the angle of incidence is

$$\theta_1 = 90 - 17.354 = 72.646$$

Snell’s law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, will tell us the refracted angle:

$$(1) \sin(72.646^\circ) = (1.33) \sin \theta_2 \implies \theta_2 = 45.861^\circ$$

This angle is also related to the distances in the drawing. In this case, the adjacent side is the 4.00 meter height and the opposite side is what we want to know. Thus,

$$\tan(45.861^\circ) = \frac{x}{4.00} \implies x = 4.1221$$

Of course, we must add the 8.00 meters to get the full length $d$:

$$d = 4.1221 + 8.00 = 12.122$$

8. A diverging lens has a focal length of -25 cm. (a) Find the image distance when an object is placed 38 cm from the lens (b) Is the image real or virtual?

**Solution**

(a) The lens equation is $1/d_o + 1/d_i = 1/f$, so

$$\frac{1}{38} + \frac{1}{d_i} = \frac{1}{-25} \implies d_i = -15.079$$

(b) Since the image distance is negative, the image is to the left side of the lens. Since the light rays are actually on the right side of the lens, the image is virtual.

9. A camper is trying to start a fire by focusing sunlight onto a piece of paper. The diameter of the sun is $1.39 \times 10^9$ meters, and its mean distance from the earth is $1.50 \times 10^{11}$ meters. The camper is using a converging lens whose focal length is 10.0 cm. (a) What is the area of the sun’s image on the paper? (b) If 0.530 watts of sunlight pass through the lens, what is the intensity of the sunlight at the paper?

**Solution**

(a) $6.74 \times 10^{-7}$ m²

(b) $7.86 \times 10^5$ W/m²
(a) In this case, the object distance is practically infinite. This means that the image distance is equal to the focal length of the lens. The magnification of the sun would therefore be

\[ m = \frac{-d_i}{d_o} = -\frac{0.100}{1.50 \times 10^{11}} = 6.6667 \times 10^{-13} \]

So the image diameter must be

\[(1.39 \times 10^9)(6.6667 \times 10^{-13}) = 9.2667 \times 10^{-4}\]

But we are asked for the area which is given by

\[ A = \pi r^2 \]

Thus,

\[ A = (\pi)(4.6333 \times 10^{-4})^2 = 6.7443 \times 10^{-7} \]

(b) The intensity is defined as \( I = P/A \), so

\[ I = \frac{0.530}{6.7443 \times 10^{-7}} = 7.8585 \times 10^5 \]

10. A converging lens \((f = 25.0 \text{ cm})\) is used to project an image of an object onto a screen. The object and the screen are 125 cm apart, and between them the lens can be placed at either of two locations. Find the two object distances.

**Solution**

Since the distance between the object and the screen is constant, we have

\[ d_o + d_i = 125 \]

In addition, we have the lens equation:

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25.0} \]

Mathematically, what we have is two equations and two unknowns. We can use the first to substitute out the \( d_i \) in the second to yield:

\[ \frac{1}{d_o} + \frac{1}{125 - d_o} = 0.0400 \]

We need common denominators, so

\[ \frac{125 - d_o}{(d_o)(125 - d_o)} + \frac{d_o}{(d_o)(125 - d_o)} = 0.0400 \]

and add:

\[ \frac{125}{(d_o)(125 - d_o)} = 0.0400 \]

then cross-multiply:

\[ 125 = (0.0400)(d_o)(125 - d_o) \]

and simplify:

\[ d_o^2 - 125d_o + 3125 = 0 \]

This quadratic equation has the solutions:

\[ d_o = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{-(125) \pm \sqrt{(-125)^2 - (4)(1)(3125)}}{(2)(1)} \]

\[ = \frac{125 \pm 55.902}{2} \]

\[ = 90.451 \text{ or } 34.549 \]