Example 1. Let \( x \) be the number of months that I have owned my chickens. Let \( V \) be a given value (in dollars). To represent the total cost of keeping chickens, I use the initial cost of $550 and the monthly cost of $19 to write \( V = 19x + 550 \). Writing an equation to represent the value of the chickens' eggs is a bit trickier. After some tinkering, I write the equation \( V = \frac{1}{2}(30(x - 3)) + \frac{1}{3}(12(x - 3)) \), which fortunately simplifies to \( V = 21x - 63 \). Solve the system of equations below to determine when the total value of the chickens' eggs will be equal to the total cost of keeping them.

\[
\begin{align*}
V &= 19x + 550 \\
V &= 21x - 63
\end{align*}
\]

[Use the substitution method]

\[
19x + 550 = 21x - 63
\]

\[
19x + 550 - 19x = 21x - 63 - 19x
\]

\[
550 = 2x - 63
\]

\[
550 + 63 = 2x - 63 + 63
\]

\[
613 = 2x
\]

\[
\frac{613}{2} = \frac{2x}{2}
\]

\[
306.5 = x
\]

Check:

\[
19(306.5) + 550 = 6373.5 \checkmark
\]

\[
21(306.5) - 63 = 6373.5 \checkmark
\]

\[
V = 19x + 550; \ x = 306.5
\]

\[
V = 19(306.5) + 550
\]

\[
= 6373.5
\]

The value of the chickens' eggs and cost will be the same after 306.5 months (or about 25.5 years). At this time, each value will be $6373.50. Realistically, this will not occur since their lifespan is 7-10 years.
Example 2. Three Clifford Bars and six Zoned Bars contain a total of 1980 calories. Four Clifford Bars and one Zoned Bar contain a total of 1170 calories. How many calories are there in one of each type of bar?

Let \( c \) be the number of calories in one Clifford Bar.
Let \( z \) be the number of calories in one Zoned Bar.

\[
\begin{align*}
3c + 6z &= 1980 \\
4c + z &= 1170
\end{align*}
\]

[Using the addition method to solve]

\[
\begin{align*}
4c + z &= 1170 \\
-6(4c + z) &= -6(1170) \\
-24c - 6z &= -7020 \\
\text{Re-written System'}.
\end{align*}
\]

\[
\begin{align*}
3c + 6z &= 1980 \\
-24c - 6z &= -7020
\end{align*}
\]

\[
\begin{align*}
3c + 6z &= 1980 \\
-24c - 6z &= -7020 \\
\hline
-21c &= -5040 \\
c &= 240
\end{align*}
\]

\[
\begin{align*}
4c + z &= 1170; c = 240 \\
4(240) + z &= 1170 \\
960 + z &= 1170 \\
z &= 210
\end{align*}
\]

One Clifford Bar contains 240 calories and one Zoned Bar contains 210 calories.
**Example 3.** Mr. Bean runs a coffee stand. He makes a blend using two different types of coffee beans. The Ethiopian beans he used sell for **$8.96 per pound** and the Columbian beans he used sell for **$4.48 per pound**. Mr. Bean made **32 pounds of the blend** and sells it at **$7.28 per pound**. How many pounds of each type of bean did Mr. Bean use in the blend?

Let \( t \) be the number of pounds of Ethiopian beans in the mix.
Let \( c \) be the number of pounds of Columbian beans in the mix.

\[
\begin{align*}
  t + c &= 32 \quad \text{(Total pounds)} \\
  8.96t + 4.48c &= 7.28(32) \quad \text{(Total cost)}
\end{align*}
\]

[Using the substitution method]

\[
\begin{align*}
  t + c &= 32 \\
  t &= 32 - c
\end{align*}
\]

Substituting into Eq. 2:

\[
\begin{align*}
  8.96(32 - c) + 4.48c &= 7.28(32) \\
  286.72 - 8.96c + 4.48c &= 232.96 \\
  286.72 - 4.48c &= 232.96 \\
  -4.48c &= -53.76 \\
  c &= 12
\end{align*}
\]

Check:

\[
\begin{align*}
  t + c &= 32; \ c = 12 \\
  t + 12 &= 32 \\
  t &= 20
\end{align*}
\]

Mr. Bean used **20 lbs of Ethiopian beans** and **12 pounds of Columbian beans**.
Example 4. Nutritional information is given for a bagel and an avocado in Table 1. How many of each should be eaten to get exactly 600 calories and 20 grams of fat?

Table 1. Nutritional Information for Bagels and Avocados

<table>
<thead>
<tr>
<th></th>
<th>Bagel</th>
<th>Avocado</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>Fat (g)</td>
<td>8</td>
<td>30</td>
</tr>
</tbody>
</table>

Let $b$ be the number of bagels eaten.
Let $a$ be the number of avocados eaten.

\[
\begin{align*}
400b + 300a &= 600 \\
8b + 30a &= 20
\end{align*}
\]

[Using the addition method]

\[
\begin{align*}
8b + 30a &= 20 \\
-10(8b + 30a) &= -10(20) \\
-80b - 300a &= -200
\end{align*}
\]

Re-written System:

\[
\begin{align*}
400b + 300a &= 600 \\
-80b - 300a &= -200
\end{align*}
\]

\[
\begin{align*}
320b &= 400 \\
b &= 1.25
\end{align*}
\]

\[
\begin{align*}
8a + 30a &= 20 \quad ; \quad b = 1.25 \\
8(1.25) + 30a &= 20 \\
10 + 30a &= 20 \\
30a &= 10 \\
a &= \frac{10}{30} \\
a &= \frac{1}{3} \approx 0.3
\end{align*}
\]

To eat exactly 600 cal. and 20 g of fat, you need to eat $1\frac{1}{3}$ bagels and $\frac{1}{3}$ avocado.

Check:

\[
\begin{align*}
400(1.25) + 300(\frac{1}{3}) &= 600 \quad \checkmark \\
8(1.25) + 30(\frac{1}{3}) &= 20 \quad \checkmark
\end{align*}
\]
Group Work 1 (Page 325 Problem 7). One Mr. Goodbar and two Mounds bars contain 780 calories. Two Mr. Goodbars and one Mounds bar contain 786 calories. Find the caloric content of each candy bar.

Let $x$ be the number of calories in one Mr. Goodbar.
Let $y$ be the number of calories in one Mounds.

System: \[
\begin{align*}
x + 2y &= 780 \\
2x + y &= 786
\end{align*}
\]

Using the substitution method:
\[x + 2y = 780\]
\[x = 780 - 2y\]

Substituting:
\[2(780 - 2y) + y = 786\]
\[1560 - 4y + y = 786\]
\[-3y = 786 - 1560\]
\[-3y = -774\]
\[y = 258\]

Finding $x$:
\[x + 2y = 780; \quad y = 258\]
\[x + 2(258) = 780\]
\[x + 516 = 780\]
\[x = 264\]

One Mr. Goodbar contains 264 calories and one Mounds bar contains 258 calories.
**Group Work 2** (Page 326 Problem 11). In a discount clothing store, all sweaters are sold at one fixed price and all shirts are sold at another fixed price. If one sweater and three shirts cost $42, while three sweaters and two shirts cost $56, find the price of one sweater and the price of one shirt.

Let \( x \) be the cost of one sweater (in dollars).
Let \( y \) be the cost of one shirt (in dollars).

System: \[
\begin{align*}
3x + 3y &= 42 \\
3x + 2y &= 56
\end{align*}
\]

Using the substitution method, we will solve Eq. 1 for \( x \) and substitute into Eq. 2:

\[
\begin{align*}
x + 3y &= 42 \\
x &= 42 - 3y
\end{align*}
\]

Substituting:

\[
\begin{align*}
3(42 - 3y) + 2y &= 56 \\
126 - 9y + 2y &= 56 \\
126 - 7y &= 56 \\
-7y &= -70 \\
y &= 10
\end{align*}
\]

\[
\begin{align*}
x + 3y &= 42; \\
x &= 42 - 3(10) \\
x &= 12
\end{align*}
\]

The cost of one sweater is \$12 and the cost of one shirt is \$10.

**Check:**

1) \( 12 + 3(10) = 42 \) ✓

2) \( 3(12) + 2(10) = 56 \) ✓
**Group Work 3** (Page 327 Problem 25). Nutritional information for macaroni and broccoli is given in Table 2. How many servings of each would it take to get exactly 14 grams of protein and 48 grams of carbohydrates?

<table>
<thead>
<tr>
<th></th>
<th>Macaroni</th>
<th>Broccoli</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protein (g)</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Carbohydrates (g)</td>
<td>16</td>
<td>4</td>
</tr>
</tbody>
</table>

Let \( m \) be the number of servings of macaroni.

Let \( b \) be the number of servings of broccoli.

\[
\begin{align*}
3m + 2b &= 14 \\
16m + 4b &= 48
\end{align*}
\]

Using the addition method:

\[
\begin{align*}
-2(3m + 2b) &= -2(14) \\
-6m - 4b &= -28
\end{align*}
\]

Re-written System:

\[
\begin{align*}
-6m - 4b &= -28 \\
16m + 4b &= 48
\end{align*}
\]

\[
\begin{align*}
-6m - 4b &= -28 \\
+ 16m + 4b &= 48
\end{align*}
\]

\[
\begin{align*}
10m &= 20 \\
m &= 2
\end{align*}
\]

\[
\begin{align*}
3m + 2b &= 14 \\
3(2) + 2b &= 14
\end{align*}
\]

\[
\begin{align*}
2b &= 8 \\
b &= 4
\end{align*}
\]

To eat 14 g of protein and 48 g of carbs, you need to eat 2 servings of macaroni and 4 servings of broccoli.

Check:

1) \[3(2) + 2(4) = 14 \checkmark\]

2) \[16(2) + 4(4) = 48 \checkmark\]