Example 7. Determine which of the following interest rates for an investment is a better deal:

- 6% compounded quarterly
  
  \[ A = A_0 \left(1 + \frac{r}{n}\right)^{nt} \]

  Effective rate:
  \[ \left(1 + \frac{0.06}{4}\right)^{4(1)} - 1 \approx 0.06136 \]
  \[ 0.136\% \]

  The investment with 6% compounded quarterly is a better deal.

- 5.95% compounded continuously
  
  \[ A = Pe^{rt} \]

  Effective rate:
  \[ e^{0.0595} - 1 \approx 0.06013 \]
  \[ 0.131\% \]

Example 8. Determine which of the following interest rates for an investment is a better deal:

- 9% compounded quarterly

  \[ A = A_0 \left(1 + \frac{0.09}{4}\right)^{4t} \]

  Effective rate:
  \[ \left(1 + \frac{0.09}{4}\right)^4 - 1 \approx 0.09308 \]
  \[ 9.308\% \]

  The account earning 9% compounded quarterly is a better deal.

- 8.95% compounded continuously

  \[ A = Pe^{0.0895t} \]

  Effective rate:
  \[ e^{0.0895} - 1 \approx 0.09363 \]
  \[ 9.363\% \]
Example 9. You invest $8,000 into an account with interest rate of 5% compounded monthly. How long will it take for the account value to reach $20,000?

\[
A = 8000 \left(1 + \frac{0.05}{12}\right)^{12t}
\]

Solve \( A = 20000 \):

\[
20000 = 8000 \left(1 + \frac{0.05}{12}\right)^{12t}
\]

\[
\frac{5}{2} = \left(1 + \frac{0.05}{12}\right)^{12t}
\]

\[
\ln\left(\frac{5}{2}\right) = \ln\left(1 + \frac{0.05}{12}\right)^{12t}
\]

\[
\ln\left(\frac{5}{2}\right) = 12t \cdot \ln\left(1 + \frac{0.05}{12}\right)
\]

\[
\frac{\ln\left(\frac{5}{2}\right)}{12 \ln\left(1 + \frac{0.05}{12}\right)} = t
\]

\[
t \approx 18.364
\]

It takes about 18.364 years for the account to reach $20,000.

Example 10. You invest $5,000 into an account that earns 2.25% interest compounded continuously. How long will it take for the account value to double?

\[
A = 5000 e^{0.0225 t}
\]

Solve \( A = 10000 \):

\[
10000 = 5000 e^{0.0225 t}
\]

\[
2 = e^{0.0225 t}
\]

\[
\ln(2) = 0.0225 t
\]

\[
\frac{\ln(2)}{0.0225} = t
\]

\[
t \approx 30.8
\]

It takes about 30.8 years for the account value to double.