A **power function** is of the form $f(x) = a_n x^n$ where $a_n$ is a real number and $n$ is a non-negative integer.

A **polynomial function** is of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and $n$ is a non-negative integer.

The **leading term** is $a_n x^n$. This determines the long-run behavior of the function.

The **degree** of the polynomial is $n$. 

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![Basic Power Functions](image1.png)

**Figure 1.** $y = x^2$

**Figure 2.** $y = x^3$

**Figure 3.** $y = x^4$

**Figure 4.** $y = x^5$

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![Basic Power Functions (close up)](image2.png)

**Figure 5.** Even Powers

**Figure 6.** Odd Powers
General Polynomial Functions

Example 1. What is the relationship between the maximum number of turning points and the degree of a polynomial function?

![Figure 7](image1.png)

![Figure 8](image2.png)

![Figure 9](image3.png)

![Figure 10](image4.png)

![Figure 11. Zeros and Their Multiplicities](image5.png)
A polynomial function $f$ has a real zero $r$ if and only if $(x - r)$ is a factor of $f(x)$.

If $r$ is a zero of **even multiplicity**, then the factor $(x - r)$ occurs an even number of times. The graph then *looks like* the graph of an even power function at that zero. Hence the function “bounces” there.

If $r$ is a zero of **odd multiplicity**, then the factor $(x - r)$ occurs an odd number of times. The graph then *looks like* the graph of an odd power function at that zero. Hence, if $(x - r)$ occurs once, the function passes “straight through” at that zero and if $(x - r)$ occurs any other odd number of time, the function “flattens” there.

**Example 2.** Graph the polynomial function defined by $f(x) = -\frac{1}{2}(x - 2)(x + 4)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.
Example 3. Graph the polynomial function defined by $f(x) = \frac{1}{4}(x + 1)^2(x + 2)(x - 5)$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

Figure 13

![Graph of the polynomial function](image-url)
Example 4. Graph the polynomial function defined by $f(x) = -\frac{1}{2}x(x + 3)(x - 2)^3$ by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

![Graph of polynomial function](image)
Example 5. Find a possible formula for the polynomial function graphed in Figure 15 by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.
Example 6. Find a possible formula for the polynomial function graphed in Figure 16 by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.

Figure 16
Example 7. Find a possible formula for the polynomial function graphed in Figure 17 by finding the following: the degree of the polynomial, the long run behavior, the maximum number of turning points, the horizontal and vertical intercepts, and the zeros and their multiplicity.