In Table 1, the 2017 federal income tax rates\(^1\) for single filing status are shown.

**Table 1. Federal Income Tax Percentage Rates for 2017 (Single Filing Status)**

<table>
<thead>
<tr>
<th>Income Amount ($\ m$, in $$)</th>
<th>Percentage of Income Taxed ($P(m)$, in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq m &lt; 9,325$</td>
<td>10</td>
</tr>
<tr>
<td>$9,325 \leq m &lt; 37,950$</td>
<td>15</td>
</tr>
<tr>
<td>$37,950 \leq m &lt; 91,900$</td>
<td>25</td>
</tr>
<tr>
<td>$91,900 \leq m &lt; 191,650$</td>
<td>28</td>
</tr>
<tr>
<td>$191,650 \leq m &lt; 416,700$</td>
<td>33</td>
</tr>
<tr>
<td>$416,700 \leq m &lt; 418,400$</td>
<td>35</td>
</tr>
<tr>
<td>$m \geq 418,400$</td>
<td>39.6</td>
</tr>
</tbody>
</table>

Notice that for each interval, the percentage of income taxed as a function of income is *constant*. If we graph each *piece* over its respective interval, we obtain the following:

**Figure 1. Graph of $y = P(m)$**

\(^1\)https://taxfoundation.org/2017-tax-brackets/
A function that is defined by different formulas on different parts of its domain is a **piecewise-defined function**.

**Example 1.** Use the piecewise-defined function $f$ defined below to answer the following.

\[
f(x) = \begin{cases} 
3x - 4 & \text{if } x \leq -2 \\
7x - 8 & \text{if } -2 < x \leq 5 \\
-11 & \text{if } x > 5 
\end{cases}
\]

(a) $f(0)$  
(b) $f(2)$  
(c) $f(-6)$  
(d) $f(8)$  
(e) $f(-2)$  
(f) $f(5)$

**Example 2.** As a prelude to graphing piecewise functions, let’s graph just a few of the “pieces.”

- Graph the linear function defined by $f(x) = -x - 2$ for values of $x$ where $-4 < x \leq -1$.
- Graph the constant function defined by $f(x) = 3$ for values of $x$ where $2 < x < 4$.
- Graph the linear function defined by $f(x) = x^2$ for values of $x$ where $x \geq -1$.

**Figure 2**  
**Figure 3**  
**Figure 4**
Example 3. Graph \( y = g(x) \) in Figure 5 for the piecewise-defined function \( g \) given below.

\[
g(x) = \begin{cases} 
-3x - 2 & \text{if } x < -1 \\
4 & \text{if } -1 \leq x < 2 \\
\frac{3}{2}x - 4 & \text{if } 2 \leq x \leq 4
\end{cases}
\]

Example 4. Graph \( y = h(x) \) in Figure 6 for the piecewise-defined function \( h \) given below.

\[
h(x) = \begin{cases} 
-x^2 + 4 & \text{if } -3 < x < 3 \\
-5 & \text{if } x \geq 4
\end{cases}
\]

Example 5. Find the formula for the piecewise-defined function \( f \) graphed in Figure 7 below.
Example 6. The graph of a piecewise function $g$ is graphed in Figure 8.

(a) State the domain and range of $g$.

(b) Evaluate $g(6)$.

(c) Evaluate $g(-2)$.

(d) Solve $g(x) = -3$.

(e) Solve $g(x) = -5$.

(f) Write the formula for the function $g$. 

\textbf{Figure 8}
Group Work 1. The graph of the piecewise-defined function $f$ is shown in Figure 9.

(a) Find the formula for this function.

(b) Find $f(1)$.

(c) Solve $f(x) = 2$.

Group Work 2. Graph the function $h$ defined below and then complete the following.

$$h(x) = \begin{cases} 
  x^2 & \text{if } -2 \leq x < 1 \\
  3 & \text{if } 1 \leq x < 3 \\
  -\frac{3}{2}(x - 5) & \text{if } 3 \leq x \leq 5 
\end{cases}$$

(a) State the domain and range of $h$.

(b) State any horizontal and vertical intercepts.

(c) State the absolute maximum of $h$ and where it occurs.
Group Work 3. When calculating your electricity bill, PGE uses the follows rates: It costs 5.124 cents per kWh for the first 250 kWh used in a month. After the first 250 kWh, it costs 6.899 cents for each additional kWh used. Let \( C(x) \) represent the monthly amount due (in dollars) for a PGE residential electricity bill where \( x \) kWh of energy were used that month.

(a) Write the formula for the piecewise-defined function \( C \).

(b) Use that formula to determine the amount due (before taxes and other fees) when you use 325 kWh of electricity in a month.

Group Work 4. The US Postal Service rates for large envelopes are given in Table 2, according to their weight.\(^2\) Graph the cost \( P \) (in dollars) of mailing a large envelope as a function of the weight \( w \) (in ounces) in Figure 11.

<table>
<thead>
<tr>
<th>Weight Not Over (in oz.)</th>
<th>Price (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>1.10</td>
</tr>
<tr>
<td>3</td>
<td>1.30</td>
</tr>
<tr>
<td>4</td>
<td>1.50</td>
</tr>
<tr>
<td>5</td>
<td>1.70</td>
</tr>
<tr>
<td>6</td>
<td>1.90</td>
</tr>
<tr>
<td>7</td>
<td>2.10</td>
</tr>
<tr>
<td>8</td>
<td>2.30</td>
</tr>
<tr>
<td>9</td>
<td>2.50</td>
</tr>
<tr>
<td>10</td>
<td>2.70</td>
</tr>
<tr>
<td>11</td>
<td>2.90</td>
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<tr>
<td>12</td>
<td>3.10</td>
</tr>
<tr>
<td>13</td>
<td>3.30</td>
</tr>
</tbody>
</table>

\(^2\)http://pe.usps.com/cpim/ftp/manuals/dmm300/Notice123.pdf