Math 111 Final Exam Review

You should be prepared to do problems similar to #1, 2, 3, 4, 5, 6, 8, 9, 10, 11 (except for rounding), 15, 16, and 17 WITHOUT a calculator.

1. Use the graph of \( y = f(x) \) in Figure 1 to answer the following. Approximate where necessary.

   (a) Evaluate \( f(-1) \).
   (b) Evaluate \( f(0) \).
   (c) Solve \( f(x) = 0 \).
   (d) Solve \( f(x) = -7 \).
   (e) Determine if \( f \) is even, odd, or neither from its graph.
   (f) State any local maximums or local minimums.
   (g) State the domain and range of \( f \).
   (h) Over what interval(s) is the function increasing?
   (i) Over what interval(s) is the function decreasing?
   (j) Over what interval(s) is the function concave up?
   (k) Over what interval(s) is the function concave down?
   (l) Find the zeros of \( f \).
   (m) Find a possible formula for this polynomial function.

![Figure 1](image1)

2. Let \( f(x) = \frac{2x - 1}{x + 2} \).

   (a) Find \( f^{-1}(x) \).
   (b) Confirm the inverse by computing \( f^{-1}(f(x)) \) and \( f(f^{-1}(x)) \).
   (c) State the domain and range of \( f \) and \( f^{-1} \).
   (d) Evaluate \( f(0) \).
   (e) Evaluate \( f^{-1}(0) \).
   (f) Solve \( f(x) = 3 \).
   (g) Algebraically determine if \( f \) is even, odd, or neither.
   (h) State any horizontal and vertical asymptotes.
   (i) State any horizontal and vertical intercepts.
   (j) Sketch a graph of \( y = f(x) \) in Figure 2.

![Figure 2](image2)
3. Let $f(x) = |x|$. For each of the following, sketch a graph of the transformation in Figure 4 and write the simplified formula for the function. Describe the order of transformations, being as specific as possible and listing them in an appropriate order.

(a) $y = -f(x)$
(b) $y = f(x + 1)$
(c) $y = 2f(x)$
(d) $y = f(x) + 3$
(e) $y = 2f(x + 1) + 3$
(f) $y = f(3x)$

Figure 3. Graph of $y = |x|$

Figure 4

4. Complete Table 1 below using the given values in the table. If any value is undefined, write “undefined.”

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$g(x)$</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>$(g \circ f)(x)$</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>$(g \cdot f)(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) + g(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{f(x)}{g(x)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g^{-1}(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Find a formula for the piecewise-defined function graphed in Figure 5 below.

\[ f(x) = \begin{cases} 
  x^2 - 4, & -3 \leq x < 0 \\
  2, & 0 < x < 1 \\
  -\frac{1}{2}x + 2, & x \geq 1 
\end{cases} \]

6. In Figure 6, graph the piecewise function defined by

\[ f(x) = \begin{cases} 
  x^2 - 4, & -3 \leq x < 0 \\
  2, & 0 < x < 1 \\
  -\frac{1}{2}x + 2, & x \geq 1 
\end{cases} \]

7. The volume, \( V(r) \) (in cubic centimeters) of a circular balloon of radius \( r \) (in centimeters) is given by \( V(r) = \frac{1}{3} \pi r^3 \). As someone blows air into the balloon, the radius of the balloon as a function of time \( t \) (in seconds) is given by \( r = g(t) = 2t \).

(a) Find and interpret \( V(3) \).
(b) Find and interpret \( g(3) \).
(c) Find and interpret \( V(g(3)) \).
(d) Find and interpret \( V(g(t)) \).
(e) Explain why \( g(V(r)) \) in nonsense.

8. Find the following for the functions \( f \), \( g \), and \( h \) defined by

\[ f(x) = \frac{2}{3x+1}, \quad g(x) = 3x^2 + 1, \quad h(x) = 2x - 5 \]

(a) \( f(g(2)) \)
(b) \( (h \circ f)(1) \)
(c) \( (h + g)(1) \)
(d) \( (g \circ g)(0) \)
(e) \( (f - g)(0) \)
(f) \( (g \cdot g)(x) \)
(g) \( (f \circ g)(x) \)
(h) \( (g \circ h)(x) \)
(i) \( (h \circ h)(x) \)

9. Find the algebraic rule (or formula) of an exponential function that passes through each pair of points:

(a) \((-1, 12)\) and \((1, 12)\)
(b) \((2, 128)\) and \((5, 2)\)

10. Write the following equations using exponents.

(a) \( \log_4 (64) = 3 \)
(b) \( \ln (\sqrt{e}) = \frac{1}{2} \)
(c) \( \log_{10} \left( \frac{1}{100} \right) = -2 \)
11. Solve the following equations. Give the exact solution and then round accurately to two decimal places. Clearly state each solution set.

(a) \(7^x - 1 = 4\)  
(b) \(e^{5x} = 10\)  
(c) \(5e^x = 10\)  
(d) \(3x^2 = 9^{x+4}\)  
(e) \(\log_4(2x + 1) = 2\)  
(f) \(\log_2(x) + \log_2(3) = \log_2(2)\)  
(g) \(\log_2(x) - \log_2(3) = \log_2(2)\)  
(h) \(2\log_5(x - 6) = \log_5(x)\)  
(i) \(\log_x(\sqrt{3}) = \frac{1}{4}\)  
(j) \(\log(1 - x) = 2 + \log(1 + x)\)

12. The temperature of a cup of tea after it was brewed can be modeled by the function \(T(t) = 100e^{-0.1t} + 68\), where \(t\) is the number of minutes since the tea was brewed and \(T(t)\) is the temperature in degrees Fahrenheit at time \(t\).

(a) Find and interpret \(f(0)\).
(b) Find and interpret \(f(10)\).
(c) Solve and interpret \(T(t) = 80\).
(d) Graph \(y = T(t)\) in your calculator. What is the horizontal asymptote?

13. Tom and Jerry make separate investments at the same time. Their respective investments can be modeled by the functions

\[T(t) = 5000(1.065)^t \quad \text{and} \quad J(t) = 4500 \left(1 + \frac{0.065}{12}\right)^{12t}\]

where \(t\) is the number of years since each investment began and \(T(t)\) and \(J(t)\) are their respective investment values in dollars.

(a) How much does Tom invest initially? How much does Jerry invest initially?
(b) What will the values of their respective investments be after 5 years?
(c) How long will it take for Jerry’s investment to double?
(d) How long will it take for their investments to be worth the same amount? How much will their respective investments be worth at this time? Solve this problem graphically using your calculator.

14. The percentage of carbon 14, \(Q\), remaining in a fossil \(t\) years since decay began can be modeled by the function

\[Q = f(t) = 100e^{-0.000124t}\]

(a) If a piece of cloth is thought to be 750 years old. What percentage of carbon 14 is expected to remain in this sample?
(b) If a fossilized leaf contains 70% of its original carbon 14, how old is the fossil?
15. Find possible equations for each polynomial function in Figure 7. List the zeros and their multiplicity, the vertical intercept, and the long-run behavior.

![Figure 7](image1.png)

(a)  
(b)  

**Figure 7**

16. Sketch a graph of \( y = f(x) \) for each polynomial function below. Also list the zeros and their multiplicity, the vertical intercept, and the long-run behavior.

(a)  \[ f(x) = -x^2(x + 4) \]  
(b)  \[ g(x) = (x - 2)(x + 1)^2(x + 2) \]

17. Find possible equations for each rational function in Figure 8 below. List the zeros and their multiplicity, any vertical asymptotes, any horizontal asymptotes, and the vertical intercept.

![Figure 8](image2.png)

(a)  

**Figure 8**