Section 3.4 The Slope-Intercept Form of the Equation of a Line (continued)

Plot \(-2x + 10y = 90\). Begin by solving the equation for \(y\).

\[
\begin{align*}
-2x + 10y &= 90 + 2x \\
10y &= 90 + 2x \\
\frac{10y}{10} &= \frac{90 + 2x}{10} \\
y &= \frac{90}{10} + \frac{2x}{10} \\
y &= 9 + \frac{1}{5}x \\
y &= \frac{1}{5}x + 9
\end{align*}
\]

Slope \( m = \frac{1}{5} \) \( \frac{\text{rise up 1}}{\text{run right 5}} \)

\( \text{y-intercept: } (0, 9) \)

\( \frac{1}{5} = \frac{-1}{-5} \) \( \frac{\text{down left 5}}{\text{up right 5}} \)
Plot $3t + 20N = 100$. Begin by solving the equation for $N$.

\[
\begin{align*}
3t + 20N - 3t &= 100 - 3t \\
20N &= 100 - 3t \\
\frac{20N}{20} &= \frac{100 - 3t}{20} \\
N &= \frac{100}{20} - \frac{3t}{20} \\
N &= 5 - \frac{3t}{20} \\
N &= -\frac{3t}{20} + 5
\end{align*}
\]

Slope $m = -\frac{3}{20}$

down 3, right 20
(or up 3, left 20)

$N$-intercept $(0, 5)$

Write an equation in the form $y = mx + b$ of the line with $y$-intercept $(0, -4)$ that is parallel to the line whose equation is $2x + 4y = 7$.

Solve $2x + 4y = 7$ for $y$.

\[
\begin{align*}
2x + 4y - 2x &= 7 - 2x \\
4y &= 7 - 2x \\
\frac{4y}{4} &= \frac{7 - 2x}{4} \\
y &= \frac{7}{4} - \frac{2x}{4} \\
y &= \frac{7}{4} - \frac{1}{2}x
\end{align*}
\]

slope: $m = -\frac{1}{2}$

The line we want has $y$-intercept $(0, -4)$.

Our line has equation $y = -\frac{1}{2}x - 4$
perpendicular lines have negative reciprocal slopes

Write an equation in the form $y = mx + b$ of the line with $y$-intercept the same as the line whose equation is $3y = 2x - 15$ and is perpendicular to the line whose equation is $y = \frac{1}{3}x + 9$.

\[
\begin{align*}
\frac{3y}{3} &= \frac{2x - 15}{3} \\
y &= \frac{2x}{3} - \frac{15}{3} \\
y &= \frac{2x}{3} - 5 &\text{our line has } y\text{-intercept } (0, -5).
\end{align*}
\]

The equation of our line is $y = -3x - 5$.

Write the equation of a decreasing line (falls from left to right) that passes through the origin and has a second point with opposite $x$- and $y$-coordinates.

\[
y = mx + b \quad b = 0
\]

$y = mx \quad m$ is negative

\[
\begin{align*}
(5, -5) \\
(-3, 3)
\end{align*}
\]

\[y = -1x\]

The equation of our line is $y = -x$. 

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Section 3.5 The Point-Slope Form of the Equation of a Line

Start with the formula for slope \( m = \frac{y_2 - y_1}{x_2 - x_1} \). Now consider a line with fixed point \((x_1, y_1)\) and an arbitrary point \((x, y)\). Applying the slope formula to these two points, we obtain \( m = \frac{y - y_1}{x - x_1} \).

Now multiply both sides of the formula by \((x - x_1)\).

\[
m(x - x_1) = \frac{y - y_1}{x - x_1} (x - x_1)
\]

\[
m(x - x_1) = (y - y_1)(x - x_1)
\]

\[
m(x - x_1) = y - y_1
\]

**Point-Slope Form of the Equation of a Line**

The point-slope form of the equation of a nonvertical line with slope \( m \) that passes through the point \((x_1, y_1)\) is \( y - y_1 = m(x - x_1) \).

Write the point-slope form and the slope intercept form of the equation of the line with slope 2 that passes through the point \((5, -1)\).

\[
m = 2 \quad (x_1, y_1) = (5, -1)
\]

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-1) = 2(x - 5) \quad \text{point-slope form}
\]

\[
y + 1 = 2(x - 5)
\]

\[
y + 1 = 2x - 10
\]

\[
y + 1 - 1 = 2x - 10 - 1 \quad \text{slope-intercept form}
\]

\[
y = 2x - 11
\]
A line passes through the points \((4, -3)\) and \((-2, 6)\). Find an equation of the line in point-slope form and then find the equation of the line in slope-intercept form.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-3)}{-2 - 4} = \frac{9}{-6} = -\frac{3}{2}
\]

Let's use \((-2, 6)\)

\[
y - y_1 = m(x - x_1)
\]

\[
y - 6 = -\frac{3}{2}(x - (-2))
\]

Point-slope form

\[
y - 6 = -\frac{3}{2}(x + 2)
\]

\[
y - 6 = -\frac{3}{2}x - 3
\]

\[
y - 6 + 6 = -\frac{3}{2}x - 3 + 6
\]

Slope-intercept form

\[
y = -\frac{3}{2}x + 3
\]

Now use the other point to write an equation of the line in point-slope form. Show that this is an equation of the same line we just found by writing it in slope-intercept form.

Let's use \((4, -3)\) now.

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-3) = -\frac{3}{2}(x - 4)
\]

Point-slope form

\[
y + 3 = -\frac{3}{2}(x - 4)
\]

\[
y + 3 = -\frac{3}{2}x - \frac{3}{2}(-4)
\]

\[
y + 3 = -\frac{3}{2}x + \frac{12}{2}
\]

\[
y + 3 = -\frac{3}{2}x + 6
\]

\[
y + 3 - 3 = -\frac{3}{2}x + 6 - 3
\]

\[
y = -\frac{3}{2}x + 3
\]

Slope-intercept form
<table>
<thead>
<tr>
<th>Form</th>
<th>What you should know</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Form (Ax + By = C)</td>
<td>Graph equations in this form using intercepts and a check point. To find the (x)-intercept, set (y = 0). To find the (y)-intercept, set (x = 0).</td>
</tr>
<tr>
<td>(y = b)</td>
<td>Graph equations in this form as horizontal lines with ((0, b)) as the (y)-intercept.</td>
</tr>
<tr>
<td>(x = a)</td>
<td>Graph equations in this form as vertical lines with ((a, 0)) as the (x)-intercept.</td>
</tr>
<tr>
<td>Slope-Intercept Form (y = mx + b)</td>
<td>Graph equations in this form using the (y)-intercept, ((0, b)), and the slope, (m). Start with this form when writing a linear equation if you know the line’s slope and (y)-intercept.</td>
</tr>
<tr>
<td>Point-Slope Form (y - y_1 = m(x - x_1))</td>
<td>Start with this form when writing a linear equation if you know the slope of the line and a point on the line other than the (y)-intercept or if you know two points on the line, neither of which is the (y)-intercept. Calculate the slope using the slope formula (m = \frac{y_2 - y_1}{x_2 - x_1}). Although you begin with point-slope form, you usually solve for (y) and convert to slope-intercept form.</td>
</tr>
</tbody>
</table>

Find the equation of the line passing through \((0, 9)\) with slope \(-\frac{1}{5}\).

\[y = mx + b\]

\[y = -\frac{1}{5}x + 9\]

Find the equation of the line passing through \((3, 0)\) and \((3, -7)\).

\[m = \frac{y_2 - y_1}{x_2 - x_1}\]

\[= \frac{-7 - 0}{3 - 3}\]

\[= \frac{-7}{0}\]

\[-\frac{7}{0}\] is undefined

Vertical lines have undefined slope.

The equation of our line is \(x = 3\).
Write an equation in slope-intercept form of the line that passes through \((5, -3)\) and is parallel to the line whose equation is \(-4x + 2y = 7\).

Parallel lines have the same slope.

Solve \(-4x + 2y = 7\) for \(y\).

\[-4x + 2y + 4x = 7 + 4x\]
\[2y = 7 + 4x\]
\[\frac{2y}{2} = \frac{7 + 4x}{2}\]
\[y = -\frac{7}{2} + 2x\]

\[y = mx + b\]
\[y = -\frac{7}{2} + 2x\]
\[m = \frac{2}{2} = 1\]

Our line has slope \(2\).

Write an equation in slope-intercept form of the line that passes through \((5, -9)\) and is perpendicular to the line whose equation is \(x + 7y = 12\).

The slopes of perpendicular lines are negative reciprocals (their product is \(-1\)).

\[x + 7y = 12\] solve for \(y\)

\[x + 7y - x = 12 - x\]
\[7y = 12 - x\]
\[\frac{7y}{7} = \frac{12 - x}{7}\]
\[y = \frac{12}{7} - \frac{x}{7}\]
\[y = -\frac{x}{7} + \frac{12}{7}\]
\[m = \frac{-1}{7}\]

Our line has slope \(7\).

\[m = 7\]
\[(5, -9)\]

\[y - y_1 = m(x - x_1)\]
\[y - (-9) = 7(x - 5)\]
\[y + 9 = 7x - 35\]
\[y + 9 - 9 = 7x - 35 - 9\]
\[y = 7x - 44\]

The equation of our line is \(y = 7x - 44\).
Write equations for the following lines.

\[ y = mx + b \]

- \[ m = -\frac{2}{1} = -2 \]
  \[ y = -2x + b \]
- \[ m = -\frac{4}{3} \]
  \[ N = \frac{4}{3}t + 4 \]
- \[ m = \frac{\text{rise}}{\text{run}} = \frac{4}{3} \]
- \[ b = 0 \]
  \[ B = -\frac{5}{2}A \]
- \[ \text{slope is undefined} \]
  \[ \chi = 5 \]
MTH 60  Supplement to Section 3.3
Comparing Slope

Table 1: The number of words
Ashley read

<table>
<thead>
<tr>
<th>t (minutes)</th>
<th>R (words)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>6</td>
<td>360</td>
</tr>
<tr>
<td>9</td>
<td>540</td>
</tr>
</tbody>
</table>

Formula for the number of words Nick has read, $R$, at time, $t$, measured in minutes

$$R = 100t$$ $m = 100$

Figure 1: Ezie’s reading

Determine who has the greatest reading speed and who reads at the slowest rate.

Table 1 has linear data. The slope is $\frac{180}{3} = 60$.

Ezie reads the fastest $\left(\frac{120 \text{ words}}{\text{min}}\right)$ and

Ashley reads the slowest $\left(\frac{60 \text{ words}}{\text{min}}\right)$
Figure 2 represents the cost to rent a car. What is the vertical-intercept of the line in Figure 2?

\((0, 50)\)

What does the vertical-intercept mean in practical terms?

The vertical-intercept tells us the car rental company has a base charge of $50 if the car travels 0 miles.

What is the slope of the line in Figure 2? Don’t forget the unit. Slopes in applied problems have units.

\[
\frac{1}{4} \text{ dollars/mile} = 0.25 \text{ dollars/mile}
\]

What does the slope mean in practical terms?

The total cost to rent the car increases at a rate of 25 cents/mile.
Figure 3 shows the height of a candle, $H$, $t$ hours after it was lit. What is the vertical-intercept of this line and what does it mean in practical terms?

The vertical intercept is $(0, 6)$ which means 0 hours since the candle was lit, the candle was 6 inches long; the candle starts out 6 inches long.

What is the horizontal intercept of this line and what does it mean in practical terms?

The horizontal intercept is $(12, 0)$ which means when the candle burns for 12 hours, it is 0 inches long; it takes 12 hours to burn out.

What is the slope of this line (include the unit) and what does it mean in practical terms?

The slope is $-\frac{1}{2}$ in/hr which means the height of the candle decreases at a rate of $\frac{1}{2}$ in/hr.