Active Filters
Topics Covered in Chapter 21

- Ideal responses
- Approximate responses
- Passive filters
- First-order stages
- VCVS unity-gain second-order low pass filters
- Higher order filters
Topics Covered in Chapter 21 (Continued)

- VCVS equal-component low pass filters
- VCVS high pass filters
- MFB bandpass filters
- Bandstop filters
- The all-pass filter
- Biquadratic and state-variable filters
Ideal filter responses

Low-pass

Bandpass

Bandstop

High-pass

All-pass
Real filter response

- **Ideal** (brickwall) filters do **not** exist.
- Real filters have an **approximate** response.
- The **attenuation** of an ideal filter is $\infty$ in the stopband.
- Real filter attenuation is $\frac{V_{out}}{V_{out(mid)}}$:
  - $3 \text{ dB} = 0.5$
  - $12 \text{ dB} = 0.25$
  - $20 \text{ dB} = 0.1$
Approximate responses

- The **passband** is identified by its low attenuation and its edge frequency
- The **stopband** is identified by its high attenuation and edge frequency
- The **order** of a filter is the number of reactive components
The order of a filter

• In an **LC** type, the order is equal to the **number** of inductors and capacitors in the filter.

• In an **RC** type, the order is equal to the **number** of capacitors in the filter.

• In an **active** type, the order is **approximately** equal to the **number** of capacitors in the filter.
Filter approximations

• **Butterworth** (maximally flat response): rolloff = 20n dB/decade where n is the order of the filter

• **Chebyshev** (equal ripple response): the number of ripples = n/2

• **Inverse Chebyshev** (rippled stopband).

• **Elliptic** (optimum transition)

• **Bessel** (linear phase shift)
Note: monotonic filters have no ripple in the stopband.
Chebyshev Inverse Butterworth

Real bandpass filter responses

Butterworth Chebyshev

Real bandpass filter responses Bessel

Real bandpass filter responses

Inverse Chebyshev Elliptic
Passive filters

- A low-pass LC filter has a resonant frequency and a $Q$
- The response is maximally flat when $Q = 0.707$
- As $Q$ increases, a peak appears in the response, centered on the resonant frequency
A second-order low-pass LC filter

\[ f_r = \frac{1}{2\pi\sqrt{LC}} \]

\[ Q = \frac{R}{X_L} \]

<table>
<thead>
<tr>
<th>L</th>
<th>C</th>
<th>( f_r )</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.55 mH</td>
<td>2.65 ( \mu )F</td>
<td>1 kHz</td>
<td>10</td>
</tr>
<tr>
<td>47.7 mH</td>
<td>531 nF</td>
<td>1 kHz</td>
<td>2</td>
</tr>
<tr>
<td>135 mH</td>
<td>187 nF</td>
<td>1 kHz</td>
<td>0.707</td>
</tr>
</tbody>
</table>

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The effect of Q on second-order response

The Butterworth response is critically damped. The Bessel response is overdamped (Q = 0.577 … not graphed). The damping factor is $\alpha$. 
First-order stages

• Have a single capacitor and one or more resistors

• Produce a **Butterworth** response because peaking is only possible in second-order or higher stages

• Can produce either a low-pass or a high-pass response
Sallen-Key second-order low-pass filter

\[ Q = 0.5 \sqrt{\frac{C_2}{C_1}} \]

\[ f_p = \frac{1}{2\pi R \sqrt{C_1 C_2}} = \text{pole frequency} \]

\[ A_v = 1 \]
Second-order responses

- Most common and easy to implement and analyze
- Butterworth: $Q = 0.707; K_c = 1$
- Bessel: $Q = 0.577; K_c = 0.786$
- Cutoff frequency: $f_c = K_c f_p$
- Peaked response: $Q > 0.707$
  * $f_0 = K_0 f_p$ (the peaking frequency)
  * $f_c = K_c f_p$ (the edge frequency)
  * $f_{3dB} = K_3 f_p$
Higher-order filters

- Cascade second-order stages to obtain even-order response.
- Cascade second-order stages plus one first-order stage to obtain odd-order response.
- The dB attenuation is cumulative.
- Filter design can be tedious and complex.
- Tables and filter-design software are used.
VCVS equal component low-pass filters

• **The Sallen-Key** equal component filters control the Q by setting the voltage gain

• **Higher Qs** are difficult to get because of component tolerance
Sallen-Key equal-component filter

As $A_v$ approaches 3, this circuit becomes impractical and may oscillate.
VCVS high-pass filters

- Have the same configuration as low-pass, except the resistors and capacitors are interchanged
- The Q values determine the K values
Sallen-Key second-order high-pass filter

\[ Q = 0.5 \sqrt{\frac{R_1}{R_2}} \]

\[ f_p = \frac{1}{2\pi C \sqrt{R_1 R_2}} \]

\[ A_v = 1 \]
MFB bandpass filters

- Low-pass and high-pass filters can be cascaded to get a bandpass filter if the $Q$ is less than 1.
- If the $Q$ is greater than 1, a narrowband rather than a wideband filter results.
Tunable MFB bandpass filter with constant bandwidth

\[ f_0 = \frac{1}{2\pi C \sqrt{2R_1(R_1 || R_3)}} \]

\[ BW = \frac{f_0}{Q} \]

\[ A_v = -1 \]

\[ Q = 0.707 \sqrt{\frac{R_1 + R_3}{R_3}} \]
Sallen-Key second-order notch filter

As $A_v$ approaches 2, this circuit becomes impractical and may oscillate.
The all-pass filter

- Passes all frequencies with no attenuation
- Controls the phase of the output signal
- Used as a phase or time-delay equalizer
First-order all-pass lag filter

\[ A_v = 1 \]
\[ f_0 = \frac{1}{2\pi RC} \]
\[ \phi = -2 \arctan \left( \frac{f}{f_0} \right) \]
First-order all-pass lead filter

\[ v_{out} = v_{in} \]

\[ R' \quad C \quad R' \]

\[ A_v = -1 \quad f_0 = \frac{1}{2\pi RC} \quad \phi = 2 \arctan \frac{f_0}{f} \]
Linear phase shift

• Required to prevent distortion of digital signals
• Constant delay for all frequencies in the passband
• Bessel design meets requirements but rolloff might not be adequate
• Designers sometimes use a non-Bessel design followed by an all-pass filter to correct the phase shift
Biquadratic filter

- Also called a TT filter
- Uses three or four op amps
- Complex but offers lower component sensitivity and easier tuning
- Has simultaneous low-pass and bandpass outputs
Biquadratic stage

\[ A_v = \frac{-R_2}{R_1} \]
\[ Q = \frac{R_2}{R_3} \]
\[ f_0 = \frac{1}{2\pi R_3 C} \]
\[ BW = \frac{1}{2\pi R_2 C} \]
State-variable filter

- Also called the **KHN** filter
- Uses three or more op amps
- When a **fourth** op amp is used, it offers easy tuning because voltage gain, center frequency, and Q are all independently tunable
State variable stage

\[ v = \frac{R_2}{R_1} + 1 \]

\[ A_v = Q = \frac{R_2}{R_1} \]

\[ f_0 = \frac{1}{2\pi RC} \]