Pre-Lab

Read through this entire lab. Perform all of your calculations (calculated values) prior to making the required circuit measurements. You may need to measure circuit component values to obtain your calculated values. All calculations should be written on a separate piece of paper (or in your lab notebook). They should be legible and written so that someone else can clearly understand your thought process. This is to demonstrate your understanding of the material, as well as aid in the troubleshooting process.

Introduction

Bandpass Filter:
In many applications, it is desired to filter all frequencies outside of a selected band. A bandpass filter (BPF) attenuates high and low frequencies, leaving the desired frequencies relatively intact.

A few common uses for BPFs are:
- Filtering music
- Tuning into a specific channel in an RF (radio) receiver
- Various uses in seismology, radar, sonar, medical, remote sensing, and any field using communications or imaging

Just like the LPF and HPF, the BPF exploits its reactive elements’ frequency-dependent reactance to achieve this. In this lab, we will explore the use of a series RLC BPF (Figure 1).

![Figure 1](image-url)
R_s is the voltage source’s series resistance and R_L is the inductor winding’s resistance.

Notice that the output voltage is across the resistor, meaning our output’s magnitude will be directly proportional to the circuit’s current. At very high frequencies, the capacitor becomes a short circuit, but the inductor becomes an open circuit, which cuts off circuit current and results in a low output voltage. At very low frequencies, the inductor becomes a short circuit, but the capacitor becomes an open circuit, which cuts off circuit current and results in a low output voltage. Only frequencies that lie within our desired bandwidth will allow significant current flow in the circuit, resulting in a larger output voltage.

This means that the filter will have two cutoff frequencies, f_1 and f_2. Just like the LPF and HPF, the BPF’s cutoff frequencies are at the half-power frequencies, meaning they occur when output voltage is about 70.7% of its maximum value. The filter’s bandwidth (BW) is the distance between the cutoff frequencies in hertz. These frequencies consist of the filter’s passband, and the filter has two stopbands. One stopband is from 0 Hz to f_1 and the other is from f_2 to infinity Hz.

Near the center of the passband is the center frequency (f_s) or resonant frequency. It is at the geometric mean of the cutoff frequencies (see equation 1).

\[ f_s = \sqrt{f_1 f_2} \]  

At the center frequency, the circuit’s total impedance becomes purely real (resistive), that is, the inductive and capacitive impedances are exact opposites and cancel each other out. At this frequency, the magnitude of the circuit’s impedance has reached a minimum, meaning the circuit’s current is at a maximum, causing V_{out} to be at its maximum.
Analysis:
Read Appendices A, B, C, and D to see how the following important BPF design specifications are mathematically related to circuit component values.

\[
\hat{H}(f) = \frac{R}{R_T + j\left(2\pi fL - \frac{1}{2\pi fC}\right)}
\]

Eqn 2: Transfer function

\[
A_v(f) = \frac{R}{\sqrt{R_T^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}
\]

Eqn 3: Circuit gain

\[
\theta(f) = \arctan\left(\frac{1}{2\pi fC} \cdot \frac{2\pi fL}{R_T}\right)
\]

Eqn 4: Phase shift

\[
f_s = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}
\]

Eqn 5: Center or Resonant Frequency

\[
\max\left(\hat{H}(f)\right) = \left(\frac{R}{R_T}\right)^{\angle 0^\circ}
\]

Eqn 6: Max circuit gain

\[
f_1 = \frac{1}{2\pi}\left[-\frac{R_T}{2L} + \frac{1}{2}\sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}}\right] \text{ Hz}
\]

Eqn 7: Lower cutoff frequency

\[
f_2 = \frac{1}{2\pi}\left[+\frac{R_T}{2L} + \frac{1}{2}\sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}}\right] \text{ Hz}
\]

Eqn 8: Upper cutoff frequency

\[
BW = \frac{R_T}{2\pi L} \text{ Hz}
\]

Eqn 9: Bandwidth, or width of passband
Required Equipment:
- Oscilloscope (Tektronix TDS 2002C)
- Function Generator
- R = 1kΩ
- C₁ = 100nF, C₂ = ?nF
- L = 100mH
- Fender Mini-Deluxe amplifier
- 6.35mm (0.25in) audio phone connector-to-BNC connector
- 3.5mm (0.25in) audio phone connector-to-breadboard cable

Part A: BPF Theory Vs. Measurements

1) Assemble the circuit in Figure 1 using C₁ for the capacitor. The 50Ω resistor is your function generator’s built in series resistance, which should be in High-Z mode, so you should have no visible 50Ω resistor in your circuit. R_L is the inductor’s winding resistance. Your voltage source will be your function generator producing a 10V_pp sinusoid with no DC offset.

2) Create an Excel table like the one in Figure 3. In order to have Excel perform your theoretical calculations later in the lab, it would be easiest to enter your measured component values without any prefixes so that Excel can interpret the cell values as numbers. R_T is the sum of R, R_L, and your function generator’s 50Ω series resistance.
3) Create an Excel table like the one in Figure 3.

<table>
<thead>
<tr>
<th>f_S (Hz)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>v_out(f_S) (V_pp)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\angle(v_out(f_S)) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f_1 (Hz)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v_out(f_1) (V_pp)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\angle(v_out(f_1)) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f_2 (Hz)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v_out(f_2) (V_pp)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\angle(v_out(f_2)) (°)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4

4) Use Equations 5, 7, and 8 to obtain theoretical values for the center and cutoff frequencies. Using Excel to do this is recommended.

5) Use Equation 6 to obtain a theoretical value for the output voltage at the center frequency. From this value, you can calculate the values for the magnitude of V_{out} at each of the cutoff frequencies. Using Excel to do this is recommended.

6) If you were to use Equation 4 to figure out the phase of the output voltage at f_S, f_1, and f_2, you would find these values to be 0°, 45°, and -45°, respectively. Fill these into the appropriate cells of your Theoretical data column.

7) With your circuit assembled, use your oscilloscope to measure your circuit’s center frequency. This will be the frequency at which the output voltage reaches its maximum value. At this measured frequency, record the magnitude and phase of the circuit’s output voltage. Record all of these values in to first 3 rows of your “Measured” column.

8) With your measured value for the maximum output voltage, use this to calculate the measured values for output voltage magnitude at each of your cutoff frequencies. Record these in the appropriate cells in your table.

9) Tune your function generator’s frequency down until you reach your measured value for output voltage magnitude at f_1. Measure and record the resulting frequency (f_1) and the phase of the output voltage. With this same method, do the same for f_2.
10) Finally, fill in the last column with percent error when comparing frequencies or voltage, and use absolute error when comparing phase angles.

Part B: Changing the Station

If this BPF were in an FM radio receiver, its passband (the frequencies between $f_1$ and $f_2$) would contain the radio station you were listening to. In other words, the BPF is letting your desired station through while filtering out all of the other stations.

Suppose you wanted to change the station. All commercial FM radio stations broadcast using the same amount of bandwidth (BW), but each have their own center frequency ($f_c$). This means that changing the station means changing the filter’s center frequency without changing its bandwidth.

1) What single component value from your circuit could you change to change the station? Look at Equations 5 and 9.

2) Using your circuit’s nominal values, solve for the value of the component you identified in part B1 that would give your circuit a new center frequency of 3,393Hz.

3) There should be a component in your lab kit that has a nominal value very close to your findings from step B2. Replace this components counterpart with the new capacitor.

4) Repeat steps A2 through A10 using your new component in place. If you used Excel as recommended in Part A, this should save you a lot of time here.

5) Use your measured component values to calculate your circuit’s bandwidth for both Part A and Part B.

Part C: Hearing the BPF’s Response

1) Obtain a Fender Mini-Deluxe amplifier along with its power supply. Plug it in. Turn the Tone and Drive knobs to their middle position. Adjust the volume to a low and reasonable level, and turn the unit off.

2) Use the 3.5mm (0.25in) audio phone connector-to-breadboard cable to connect the computer’s headphone jack to the breadboard. Use the 3.5mm (0.25in) audio phone connector-to-breadboard cable to connect Input jack of the amplifier to the computer headphone jack via the breadboard. We will be using music from the computer as the voltage source that we will be filtering.
One side should be connected to ground (the lead soldered to the bare wire) and the other side should either be connected to the left or right channel. It does not matter which one.

Play some music on the computer. It should be appropriate for class.

Turn on your amplifier and adjust the volume to a reasonable level (level 2, or so). If the song sounds distorted, you may want to turn down the volume on the computer until the distortion goes away. Leave the amplifier off when you do not need to hear the music.

3) After making sure your amplifier is playing music satisfactorily, apply your BPF from Part B to the music voltage signal and observe what it sounds like.

4) Apply your BPF from Part A (simply swap your old capacitor back in) to the music voltage signal and observe what it sounds like. How do the two filters’ outputs compare to each other?
Appendix A: Derivation of the Circuit’s Transfer Function and its Magnitude and Phase

We start off by finding output voltage with the voltage divider rule.

\[ \hat{V}_{out} = \hat{V}_{in} \left( \frac{R}{R + R_S + R_L + j2\pi fL} \right) = \hat{V}_{in} \left( \frac{R}{R_T + j2\pi fL} \right) \]

where \( R_T = R + R_S + R_L \)

\[ \hat{V}_{in} \left( \frac{R}{R_T + j \left( 2\pi fL - \frac{1}{2\pi fC} \right)} \right) = \hat{V}_{in} \left( \frac{R}{R_T} \right) \]

\[ \hat{H}(f) = \frac{\hat{V}_{out}}{\hat{V}_{in}} = \frac{\hat{V}_{in} \left( \frac{R}{R_T + j \left( 2\pi fL - \frac{1}{2\pi fC} \right)} \right)}{\hat{V}_{in}} = \frac{R}{R_T + j \left( 2\pi fL - \frac{1}{2\pi fC} \right)} \]

\[ \hat{H}(f) = \frac{R}{R_T + j \left( 2\pi fL - \frac{1}{2\pi fC} \right)} \quad \text{Eqn 2} \]
From this, we can find the equation for circuit gain and phase shift as a function of frequency.

\[ A_v(f) = \frac{R}{R_r + j \left( \frac{2\pi f L}{2\pi f C} \right)} = \frac{R}{\sqrt{R_r^2 + \left( \frac{2\pi f L}{2\pi f C} \right)^2}} \]

\[ A_v(f) = \frac{R}{R_r^2 + \left( \frac{2\pi f L}{2\pi f C} \right)^2} \quad \text{Eqn 3} \]

\[ \theta(f) = \angle \left( \frac{R}{R_r + j \left( \frac{2\pi f L}{2\pi f C} \right)} \right) = \angle(R) - \angle \left( R_r + j \left( \frac{2\pi f L}{2\pi f C} \right) \right) \]

\[ = 0 - \arctan \left( \frac{2\pi f L}{2\pi f C} \right) = - \arctan \left( \frac{2\pi f L}{2\pi f C} \right) = \arctan \left( \frac{2\pi f L}{2\pi f C} \right) \]

\[ \theta(f) = \arctan \left( \frac{1}{2\pi f C} - \frac{2\pi f L}{R_r} \right) \quad \text{Eqn 4} \]
Appendix B: Derivation of the Resonant Frequency

Remember that at the resonant or center frequency \( f_s \), the total impedance of the circuit is purely resistive. The center frequency is a very important circuit design specification, so its mathematical relationship to circuit component values is derived below. It begins with Equation 2.

\[
\hat{H}(f) = \frac{R}{R_f + j \left( 2\pi f L - \frac{1}{2\pi f C} \right)}
\]

when \( \left( 2\pi f L - \frac{1}{2\pi f C} \right) = 0 \), then the \( V_{out} \) reaches its maximum

\[
2\pi f_s L - \frac{1}{2\pi f_s C} = 0
\]

\[
2\pi f_s L = \frac{1}{2\pi f_s C}
\]

\[
f_s^2 = \frac{1}{(2\pi)^2 LC}
\]

\[
f_s = \frac{1}{2\pi \sqrt{LC}} \text{ Hz} \quad \text{Eqn 5}
\]
Appendix C: Resonant Frequency as a Function of Circuit Component Values

The maximum output of the circuit occurs at the center or resonant frequency.

\[
\max\left(\hat{H}(f)\right) = \hat{H}(f_r) = \frac{R}{R_T + j\left(2\pi f_r L - \frac{1}{2\pi f_r C}\right)}
\]

\[
= \frac{R}{R_T + j\left(2\pi \left(\frac{1}{2\pi \sqrt{LC}}\right)L - \frac{1}{2\pi \left(\frac{1}{2\pi \sqrt{LC}}\right)}C\right)}
\]

\[
= \frac{R}{R_T + j\left(\frac{L}{\sqrt{LC}} - \frac{1}{\sqrt{LC}}\right)} = \frac{R}{R_T + j\left(\frac{L}{\sqrt{LC}} - \frac{\sqrt{LC}}{C}\right)} = \frac{R}{R_T + j\left(\frac{L^2}{LC} - \frac{LC}{C^2}\right)}
\]

\[
= \frac{R}{R_T + j\left(\sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}}\right)} = \frac{R}{R_T + j(0)} = \frac{R}{R_T} = \left(\frac{R}{R_T}\right) \angle 0^\circ
\]

\[
\max\left(\hat{H}(f)\right) = \left(\frac{R}{R_T}\right) \angle 0^\circ \quad \text{Eqn 6}
\]
Appendix D: Cutoff Frequencies and Bandwidth as a Function of Circuit Component Values

Another important BPF design specification is the center frequencies. Their mathematical relationship with circuit values is derived below. Remember that, by definition, $V_{out}$ is at $\frac{1}{\sqrt{2}}$ of its maximum value of $\frac{R}{R_t}$ at the frequencies $f_1$ and $f_2$. We will start off using Equations 3 and 6.

$$A_v(f) = \left(\frac{1}{\sqrt{2}}\right) \left(\text{max value of } V_{out}\right)$$

solve for the two frequencies at which this relationship is possible (at $f = f_1$ and $f_2$)

$$\frac{R}{\sqrt{R_t^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}} = \left(\frac{1}{\sqrt{2}}\right) \left(\frac{R}{R_t}\right)$$

using $\omega = 2\pi f$

$$\frac{1}{\sqrt{R_t^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{R_t \sqrt{2}}$$

$$\sqrt{R_t^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R_t \sqrt{2}$$

$$R_t^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R_t^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R_t^2$$

$$\omega L - \frac{1}{\omega C} = \pm R_t$$

$$\omega L - \frac{1}{\omega C} = \pm R_t$$

$$\omega^2 L - \frac{1}{C} \pm \omega R_t = 0$$

$$\omega^2 (L) + \omega (\pm R_t) + \left(-\frac{1}{C}\right) = 0$$
using the quadratic equation, \( \omega_1, \omega_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

where \( a = L, \ b = \pm R_T, \ c = -\frac{1}{C} \)

\[
\omega_1, \omega_2 = \frac{\pm (\pm R_T) \pm \sqrt{(-R_T)^2 - 4L\left(-\frac{1}{C}\right)}}{2L} = \frac{\pm R_T \pm \sqrt{R_T^2 + \frac{4L}{C}}}{2L}
\]

Recognizing that if the "±" was negative, we would get a negative number for frequency. So "±" will be changed to "+".

\[
\frac{\pm R_T + \sqrt{R_T^2 + \frac{4L}{C}}}{2L} = \frac{R_T}{2L} + \sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}}
\]

\[
f_1, f_2 = \left(\frac{1}{2\pi}\right) \omega = \frac{1}{2\pi} \left[ \pm \frac{R_T}{2L} + \frac{1}{2} \sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}} \right]
\]

\[
f_1 = \frac{1}{2\pi} \left[ -\frac{R_T}{2L} + \frac{1}{2} \sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}} \right] \text{ Hz} \quad \text{Eqn 7}
\]

\[
f_2 = \frac{1}{2\pi} \left[ +\frac{R_T}{2L} + \frac{1}{2} \sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}} \right] \text{ Hz} \quad \text{Eqn 8}
\]

\[
BW = f_2 - f_1 = \frac{1}{2\pi} \left[ +\frac{R_T}{2L} + \frac{1}{2} \sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}} \right] \left( -\frac{1}{2\pi} \left[ -\frac{R_T}{2L} + \frac{1}{2} \sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}} \right] \right) - \frac{1}{2\pi} \left[ -\frac{R_T}{2L} + \frac{1}{2} \sqrt{\left(\frac{R_T}{L}\right)^2 + \frac{4}{LC}} \right]
\]

\[
= \frac{1}{2\pi} \left[ \frac{R_T}{2L} + \frac{R_T}{2L} \right] = \frac{1}{2\pi} \left[ \frac{R_T}{2L} \right] = \frac{R_T}{2\pi L}
\]

\[
BW = \frac{R_T}{2\pi L} \text{ Hz} \quad \text{Eqn 9}
\]