

Key Concepts: Division of Rational Expression
Complex Rational Expressions

Division of rational expressions

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \div \frac{C}{D} \quad \text{assuming } B \neq 0, C \neq 0, \text{ and } D \neq 0$$

$$= \frac{A}{B} \times \frac{D}{C}$$

Example 1

Completely simplify $\frac{\frac{2x-3}{x^2-4}}{\frac{x-5}{x+2}}$.

$$\begin{aligned} \frac{\frac{2x-3}{x^2-4}}{\frac{x-5}{x+2}} &= \frac{2x-3}{x^2-4} \cdot \frac{x+2}{x-5} \\ &= \frac{(2x-3)(x+2)}{(x+2)(x-2)(x-5)} \\ &= \frac{x+2}{x+2} \cdot \frac{2x-3}{(x-2)(x-5)} \\ &= \frac{2x-3}{(x-2)(x-5)} \end{aligned}$$

Complex Fractions

A *complex* fraction is any fraction whose numerator *or* denominator contains another fraction.

Simplifying complex rational expressions – Strategy A

1. Simplify (add and/or subtract, as necessary) the expressions in the numerator and denominator.
2. Reciprocate the expression in the denominator and multiply with the expression in the numerator.
3. Simplify the remaining expression.

Example 2

Completely simplify $f(x) = \frac{\frac{2}{x+3} - \frac{4}{x-1}}{\frac{x+2}{x+3}}$. Verify the simplified formula for $x = -1$.

$$\begin{aligned}
 f(x) &= \frac{\frac{2}{x+3} - \frac{4}{x-1}}{\frac{x+2}{x+3}} \\
 &= \frac{\frac{2}{x+3} \cdot \frac{x-1}{x-1} - \frac{4}{x-1} \cdot \frac{x+3}{x+3}}{\frac{x+2}{x+3}} \\
 &= \frac{\frac{2(x-1) - 4(x+3)}{(x+3)(x-1)}}{\frac{x+2}{x+3}} \\
 &= \frac{2x-2-4x-12}{(x+3)(x-1)} \cdot \frac{x+3}{x+2} \\
 &= \frac{(-2x-14)(x+3)}{(x+3)(x-1)(x+2)} = \frac{x+3}{x+2} \cdot \frac{-2(x+7)}{(x-1)(x+2)} \\
 &= -\frac{2(x+7)}{(x-1)(x+2)}
 \end{aligned}$$

Using original formula

$$\begin{aligned}
 f(-1) &= \frac{\frac{2}{-1+3} - \frac{4}{-1-1}}{\frac{-1+2}{-1+3}} \\
 &= \frac{\frac{1}{2} - \frac{4}{-2}}{\frac{1}{2}} \\
 &= \frac{3}{1} \times \frac{2}{2} \\
 &= 6
 \end{aligned}$$

using answer

$$\begin{aligned}
 f(-1) &= -\frac{2(6)}{(-2)(1)} \\
 &= 6
 \end{aligned}$$

Example 3

Completely simplify $\frac{\frac{3}{y^2-4} - \frac{y}{4-y^2}}{\frac{y}{y-2} - \frac{3}{2-y}}$. Verify the simplified formula for $y=3$.

$$\frac{\frac{3}{y^2-4} - \frac{y}{4-y^2}}{\frac{y}{y-2} - \frac{3}{2-y}} = \frac{\frac{3}{y^2-4} - \frac{y}{4-y^2} \cdot \frac{-1}{-1}}{\frac{y}{y-2} - \frac{3}{2-y} \cdot \frac{-1}{-1}}$$

$$= \frac{\frac{3}{y^2-4} - \frac{-y}{y^2-4}}{\frac{y}{y-2} - \frac{-3}{y-2}}$$

$$= \frac{\frac{3+y}{y^2-4}}{\frac{y+3}{y-2}}$$

$$= \frac{y+3}{y^2-4} \cdot \frac{y-2}{y+3}$$

$$= \frac{(y+3)(y-2)}{(y+2)(y-2)(y+3)}$$

$$= \frac{y+3}{y+3} \cdot \frac{y-2}{y-2} \cdot \frac{1}{y+2}$$

$$= \frac{1}{y+2}$$

Check

From original
($y=3$)

$$\frac{\frac{3}{5} - \frac{3}{-5}}{\frac{3}{1} - \frac{3}{-1}}$$

$$= \frac{\frac{6}{5}}{\frac{6}{1}} = \frac{6}{5} \times \frac{1}{6} = \frac{1}{5}$$

From newbie
($y=3$)

$$\frac{1}{3+2} = \frac{1}{5} \checkmark$$

Example 4

Completely simplify $\frac{\frac{2}{t+h} - \frac{2}{t}}{h}$.

$$\begin{aligned}
 \frac{\frac{2}{t+h} - \frac{2}{t}}{h} &= \frac{\frac{2}{t+h} \cdot \frac{t}{t} - \frac{2}{t} \cdot \frac{t+h}{t+h}}{h} \\
 &= \frac{\frac{2t - 2(t+h)}{t(t+h)}}{\frac{h}{1}} \\
 &= \frac{2t - 2t - 2h}{t(t+h)} \cdot \frac{1}{h} \\
 &= \frac{-2h}{t(t+h)h} \\
 &= \frac{h}{h} \cdot \frac{-2}{t(t+h)} \\
 &= -\frac{2}{t(t+h)}
 \end{aligned}$$

Example 5

Completely simplify $\frac{\frac{2}{x+1} + \frac{4}{(x+1)^2}}{\frac{2}{(x+1)(x-2)}}$. Verify your formula when $x = 1$.

$$\begin{aligned}
 \frac{\frac{2}{x+1} + \frac{4}{(x+1)^2}}{\frac{2}{(x+1)(x-2)}} &= \frac{\frac{2}{x+1} \cdot \frac{x+1}{x+1} + \frac{4}{(x+1)^2}}{\frac{2}{(x+1)(x-2)}} \\
 &= \frac{\frac{2(x+1) + 4}{(x+1)(x+1)}}{\frac{2}{(x+1)(x-2)}} \\
 &= \frac{2x+2+4}{(x+1)(x+1)} \cdot \frac{(x+1)(x-2)}{2} \\
 &= \frac{(2x+6)(x+1)(x-2)}{2(x+1)(x+1)} \\
 &= \frac{x+1}{x+1} \cdot \frac{2(x+3)(x-2)}{2(x+1)} \\
 &= \frac{(x+3)(x-2)}{x+1}
 \end{aligned}$$

$$\sqrt[3]{3020}$$

Example 6

Completely simplify $\frac{\frac{1}{x-4} + \frac{1}{x+4}}{1 - \frac{1}{x+4}}$

$$\begin{aligned}
 \frac{\frac{1}{x-4} + \frac{1}{x+4}}{1 - \frac{1}{x+4}} &= \frac{\frac{1}{x-4} \cdot \frac{x+4}{x+4} + \frac{1}{x+4} \cdot \frac{x-4}{x-4}}{1 \cdot \frac{x+4}{x+4} - \frac{1}{x+4}} \\
 &= \frac{\frac{x+4 + x-4}{(x-4)(x+4)}}{\frac{x+4-1}{x+4}} \\
 &= \frac{\frac{2x}{(x-4)(x+4)}}{\frac{x+3}{x+4}}
 \end{aligned}$$

Check $x=5$
Original formula

$$\begin{aligned}
 \frac{\frac{1}{1} + \frac{1}{9}}{1 - \frac{1}{9}} &= \frac{10/9}{8/9} \\
 &= \frac{10}{9} \cdot \frac{9}{8} \\
 &= \frac{5}{4}
 \end{aligned}$$

new formula

$$\frac{10}{(1)(8)} = \frac{5}{4} \checkmark$$

$$\begin{aligned}
 &= \frac{2x}{(x-4)(x+4)} \cdot \frac{x+4}{x+3} \\
 &= \frac{x+4}{x+4} \cdot \frac{2x}{(x-4)(x+3)} \\
 &= \frac{2x}{(x-4)(x+3)}
 \end{aligned}$$