Key Concepts: LCDs for Rational Expressions
Adding and subtracting rational expressions

Finding the LCD (least common dominator) of several rational expressions

1. Completely factor each denominator.
2. If necessary, multiply one or more expression by $\frac{-1}{-1}$ so that no pair of expressions contain opposite factors in their denominators.
3. Establish the least common multiple of the denominators' numeric factors; this is a factor of the LCD.
4. A variable factor that occurs in *any* of the denominators is a factor of the LCD; the exponent on that factor is the greatest exponent that occurs on the factor in *any* <u>*one*</u> denominator.

Please note that in any given fraction there may be common factors between the numerator and denominator. It is probably best to go ahead and eliminate these factors before determining the LCD. The entire reason you want to establish the *least* common dominator is that it will result in the *least* amount of residual work once you combine the expressions.

<u>Example 1</u>
Find the LCD for each sum or difference.

$$\frac{5}{x+2} - \frac{3}{x-2}$$ The LCD is $(x+2)(x-2)$

$$\frac{7}{x^2-2x-3} + \frac{5}{x^2+x-12}$$

$$\frac{7}{x^2-2x-3} + \frac{5}{x^2+x-12} = \frac{7}{(x-3)(x+1)} + \frac{5}{(x+4)(x-3)}$$

The LCD is $(x-3)(x+1)(x+4)$

$$(4-y)(-1) = -4+y$$
$$= y+(-4)$$
$$= y-4$$

$$\frac{3}{x+2}+\frac{5}{x^2+4x+4}$$

$$\frac{3}{x+2}+\frac{5}{x^2+4x+4} = \frac{3}{x+2}+\frac{5}{(x+2)(x+2)}$$
$$= \frac{3}{x+2}+\frac{5}{(x+2)^2}$$

The LCD is $(x+2)^2$

$$\frac{y-2}{10y-40}-\frac{7}{48-12y}$$

$$\frac{y-2}{10y-40}-\frac{7}{48-12y} = \frac{y-2}{10(y-4)}-\frac{7}{12(4-y)}\cdot\frac{-1}{-1}$$
$$= \frac{y-2}{10(y-4)}-\frac{-7}{12(y-4)}$$
$$= \frac{y-2}{10(y-4)}+\frac{7}{12(y-4)}$$

The LCD is $60(y-4)$

$$\frac{4a^2bc^3}{10a^7b^6}+\frac{3ac}{5a^{12}b^9c^7}$$

$$\frac{4a^2bc^3}{10a^7b^6}+\frac{3ac}{5a^{12}b^9c^7} = \frac{2}{2}\cdot\frac{a^2}{a^2}\cdot\frac{b}{b}\cdot\frac{2c^3}{5a^5b^5}+\frac{a}{a}\cdot\frac{c}{c}\cdot\frac{3}{5a^{11}b^9c^6}$$
$$= \frac{2c^3}{5a^5b^5}+\frac{3}{5a^{11}b^9c^6}$$

The LCD is $5a^{11}b^9c^6$

Adding and subtracting rational expressions

1. Establish the LCD
2. Multiply each expression, if necessary, by the form of 1 that builds that fraction's denominator up to the LCD.
3. Add or subtract the numerators over the LCD
4. *Do not expand the denominator … EVER!!*
5. Completely expand the numerator.
6. Completely factor the numerator.
7. Eliminate common factors between the numerator and denominator. Make sure that you remember to state the necessary domain restrictions.

Example 4

Completely simplify each rational expression making sure to state any necessary domain restrictions.

$$\frac{x+2}{x-4}-\frac{4}{x-5}$$

$$\frac{x+2}{x-4}-\frac{4}{x-5}=\frac{x+2}{x-4}\cdot\frac{x-5}{x-5}-\frac{4}{x-5}\cdot\frac{x-4}{x-4}$$

$$=\frac{(x+2)(x-5)-4(x-4)}{(x-4)(x-5)} \Leftarrow \text{The LCD}$$

$$=\frac{x^2-3x-10-4x+16}{(x-4)(x-5)}$$

$$=\frac{x^2-7x+6}{(x-4)(x-5)}$$

$$=\frac{(x-6)(x-1)}{(x-4)(x-5)}$$

$$\frac{2}{t^2-4}-\frac{t+4}{t-2}$$

$$\frac{2}{t^2-4}-\frac{t+4}{t-2}=\frac{2}{(t+2)(t-2)}-\frac{t+4}{t-2}$$
$$=\frac{2}{(t+2)(t-2)}-\frac{t+4}{t-2}\cdot\frac{t+2}{t+2}$$
$$=\frac{2-(t+4)(t+2)}{(t+2)(t-2)}$$
$$=\frac{2-(t^2+6t+8)}{(t+2)(t-2)}$$
$$=\frac{2-t^2-6t-8}{(t+2)(t-2)}$$
$$=\frac{-t^2-6t-6}{(t+2)(t-2)}$$
$$=-\frac{t^2+6t+6}{(t+2)(t-2)}$$

$$\frac{x+y}{x-y}-\frac{4xy}{x^2-y^2}$$

$$\frac{x+y}{x-y}-\frac{4xy}{x^2-y^2}=\frac{x+y}{x-y}-\frac{4xy}{(x+y)(x-y)}$$
$$=\frac{x+y}{x-y}\cdot\frac{x+y}{x+y}-\frac{4xy}{(x+y)(x-y)}$$
$$=\frac{(x+y)(x+y)-4xy}{(x-y)(x+y)}$$
$$=\frac{x^2+2xy+y^2-4xy}{(x-y)(x+y)}$$
$$=\frac{x^2-2xy+y^2}{(x-y)(x+y)}$$
$$=\frac{(x-y)(x-y)}{(x-y)(x+y)}$$
$$=\frac{x-y}{x-y}\cdot\frac{x-y}{x+y}$$
$$=\frac{x-y}{x+y}\ ;\ y\neq x$$

$$(3-y)(-1) = -3+y$$
$$= y+(-3)$$
$$= y-3$$

$\frac{y}{3-y}+\frac{6}{2y-6}$

$$\frac{y}{3-y}+\frac{6}{2y-6} = \frac{y}{3-y}+\frac{6}{2(y-3)}$$
$$= \frac{y}{3-y}+\frac{3}{y-3}$$
$$= \frac{y}{3-y}\cdot\frac{-1}{-1}+\frac{3}{y-3}$$
$$= \frac{-y}{y-3}+\frac{3}{y-3}$$
$$= \frac{-y+3}{y-3}$$
$$= \frac{-1(y-3)}{y-3}$$
$$= -1;\ y\neq 3$$

$x-\frac{3}{x-2}$

$$x-\frac{3}{x-2} = \frac{x}{1}\cdot\frac{x-2}{x-2}-\frac{3}{x-2}$$
$$= \frac{x(x-2)-3}{x-2}$$
$$= \frac{x^2-2x-3}{x-2}$$
$$= \frac{(x-3)(x+1)}{x-2}$$

Bonus info.
The vertical asymptote on $y=x-\frac{3}{x-2}$ is $x=2$

Factor of -10

10, -1 nope

5, -2 yep

$$2t^2+3t-5 = 2t^2+5t-2t-5$$
$$= t(2t+5)-1(2t+5)$$
$$= (2t+5)(t-1)$$

$$\frac{3}{2t^2+3t-5}-\frac{2}{t^2-1}-\frac{1}{2t^2+7t+5}$$

$$\frac{3}{2t^2+3t-5}-\frac{2}{t^2-1}-\frac{1}{2t^2+7t+5}$$

$$=\frac{3}{(2t+5)(t-1)}-\frac{2}{(t+1)(t-1)}-\frac{1}{(2t+5)(t+1)}$$

$$=\frac{3}{(2t+5)(t-1)}\cdot\frac{t+1}{t+1}-\frac{2}{(t+1)(t-1)}\cdot\frac{2t+5}{2t+5}-\frac{1}{(2t+5)(t+1)}\cdot\frac{t-1}{t-1}$$

$$=\frac{3(t+1)-2(2t+5)-1(t-1)}{(2t+5)(t-1)(t+1)}$$

$$=\frac{3t+3-4t-10-t+1}{(2t+5)(t-1)(t+1)}$$

$$=\frac{-2t-6}{(2t+5)(t-1)(t+1)}$$

$$=-\frac{2(t+3)}{(2t+5)(t-1)(t+1)}$$

$t-1\neq 0$

$t+1\neq 0$

$2t+5\neq 0$

$t\neq 1$

$t\neq -1$

$t\neq -5/2$

Bonus

The domain is $\left(-\infty,-\frac{5}{2}\right)\cup\left(-\frac{5}{2},-1\right)\cup(-1,1)\cup(1,\infty)$