Key Concepts: Solving Quadratic Equations using the Square Root Principle
Completing the Square
***i* yi yi!**

The Square Root Principle

For $k \neq 0$, there are two solutions to $u^2 = k$: $\sqrt{k}$ and $-\sqrt{k}$.

The only solution to $u^2 = 0$ is 0.

Examples

Find all solutions to each equation – completely simplify the solutions

$(2t+1)^2 = 9$

$$(2t+1)^2 = 9$$

$2t+1 = \sqrt{9}$ OR $2t+1 = -\sqrt{9}$

$2t+1 = 3$ or $2t+1 = -3$

$2t = 2$ OR $2t = -4$

$t = 1$ OR $t = -2$

The solutions are 1 and -2.

$(6x-4)^2 = 72$

$$(6x-4)^2 = 72$$

$6x-4 = \sqrt{72}$ OR $6x-4 = -\sqrt{72}$

$6x-4 = 6\sqrt{2}$ or $6x-4 = -6\sqrt{2}$

$6x = 4+6\sqrt{2}$ OR $6x = 4-6\sqrt{2}$

$x = \dfrac{4+6\sqrt{2}}{6}$ OR $x = \dfrac{4-6\sqrt{2}}{6}$

$x = \dfrac{\cancel{2}(2+3\sqrt{2})}{\cancel{2}\cdot 3}$ or $x = \dfrac{\cancel{2}(2-3\sqrt{2})}{\cancel{2}\cdot 3}$

$x = \dfrac{2+3\sqrt{2}}{3}$ OR $x = \dfrac{2-3\sqrt{2}}{3}$

The solutions are $\dfrac{2+3\sqrt{2}}{3}$ and $\dfrac{2-3\sqrt{2}}{3}$.

$x^2 - 8x + 16 = 12$

$x^2 - 8x + 16 = 12$

$x^2 - 8x = -4$

$x^2 - 8x + \underline{16} = -4 + \underline{16}$

$(x-4)^2 = 12$

$x - 4 = \sqrt{12}$ or $x - 4 = -\sqrt{12}$

$x - 4 = 2\sqrt{3}$ or $x - 4 = -2\sqrt{3}$

$x = 4 + 2\sqrt{3}$ or $x = 4 - 2\sqrt{3}$

The solutions are $4 + 2\sqrt{3}$ and $4 - 2\sqrt{3}$.

$t^2 - 6t + 2 = 0$

$t^2 - 6t + 2 = 0$

$t^2 - 6t = -2$

$t^2 - 6t + \underline{9} = -2 + \underline{9}$

$(t-3)^2 = 7$

$t - 3 = \sqrt{7}$ or $t - 3 = -\sqrt{7}$

$t = 3 + \sqrt{7}$ or $t = 3 - \sqrt{7}$

The solutions are $3 + \sqrt{7}$ and $3 - \sqrt{7}$.

$y^2 + 20y + 80 = 0$

$y^2 + 20y + 80 = 0$

$y^2 + 20y = -80$

$y^2 + 20y \underline{+100} = -80 \underline{+100}$

$(y+10)^2 = 20$

$y + 10 = \sqrt{20}$ or $y + 10 = -\sqrt{20}$

$y + 10 = 2\sqrt{5}$ or $y + 10 = -2\sqrt{5}$

$y = -10 + 2\sqrt{5}$ or $y = -10 - 2\sqrt{5}$

The solutions are $-10 + 2\sqrt{5}$ and $-10 - 2\sqrt{5}$

Examples

Find the x-intercepts on each parabola.

$\cancel{y = x^2 - 7x + 7}$ $y = 2x^2 - 20x + 8$

The x-intercepts occur where $y = 0$.

$$2x^2 - 20x + 8 = 0$$

$$2x^2 - 20x = -8$$

$$\frac{2x^2 - 20x}{2} = \frac{-8}{2}$$

$$x^2 - 10x = -4$$

$$x^2 - 10x + \underline{25} = -4 + \underline{25}$$

$$(x-5)^2 = 21$$

$x - 5 = \sqrt{21}$ or $x - 5 = -\sqrt{21}$

$x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

The x-intercepts are

$(5+\sqrt{21}, 0)$ and $(5-\sqrt{21}, 0)$.

$\cancel{g(x) = x^2 + 4x + 7}$

$y = 3x^2 - 18x + 27$

The x-intercepts occur where $y = 0$

$$3x^2 - 18x + 27 = 0$$

$$3x^2 - 18x = -27$$

$$\frac{3x^2 - 18x}{3} = \frac{-27}{3}$$

$$x^2 - 6x = -9$$

$$x^2 - 6x + \underline{9} = -9 + \underline{9}$$

$$(x-3)^2 = 0$$

$$x - 3 = 0$$

$$x = 3$$

The only x-intercept is $(3, 0)$.

Bonus +3.5

$$\sqrt[6]{y^{13}} = \sqrt[6]{y^{12}}\sqrt[6]{y} = y^2\sqrt[6]{y}$$

i Carumba!

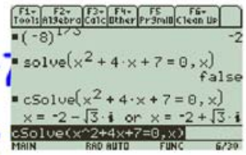
The positive square root of -1 is defined to be the number i.

$\sqrt{-1} = i$ and $-\sqrt{-1} = -i$

Example

Find all solutions to $g(x) = 0$ where $g(x) = x^2 + 4x + 7$; completely simplify the solutions.

$$g(x) = 0$$
$$x^2 + 4x + 7 = 0$$
$$x^2 + 4x = -7$$
$$x^2 + 4x + \underline{4} = -7 + 4$$
$$(x+2)^2 = -3$$
$$x + 2 = \sqrt{-3} \text{ or } x + 2 = -\sqrt{-3}$$
$$x + 2 = \sqrt{3}\sqrt{-1} \text{ or } x + 2 = -\sqrt{3}\sqrt{-1}$$



$$x + 2 = \sqrt{3}\,i \text{ or } x + 2 = -\sqrt{3}\,i$$
$$x = -2 + \sqrt{3}\,i \text{ or } x = -2 - \sqrt{3}\,i$$

The solutions $-2 + i\sqrt{3}$ and $-2 - i\sqrt{3}$

Example

Find all solutions to $(4y - 12)^2 = -18$; completely simplify the solutions.

$$(4y - 12)^2 = -18$$
$$4y - 12 = \sqrt{-18} \text{ or } 4y - 12 = -\sqrt{-18}$$
$$4y - 12 = \sqrt{9}\sqrt{2}\sqrt{-1} \text{ or } 4y - 12 = -\sqrt{9}\sqrt{2}\sqrt{-1}$$
$$4y - 12 = 3\sqrt{2}\,i \text{ or } 4y - 12 = -3\sqrt{2}\,i$$
$$4y = 12 + 3i\sqrt{2} \text{ or } 4y = 12 - 3i\sqrt{2}$$
$$y = \frac{12 + 3i\sqrt{2}}{4} \text{ or } y = \frac{12 - 3i\sqrt{2}}{4}$$
$$y = \frac{12}{4} + \frac{3i\sqrt{2}}{4} \text{ or } y = \frac{12}{4} - \frac{3i\sqrt{2}}{4}$$

The solutions are $3 \pm \frac{3\sqrt{2}}{4}i$

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2$

$= -1$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ if and only if a and b are not both negative.

True or False

$\underbrace{\sqrt{-4}\,\sqrt{-9} = \sqrt{(-4)(-9)}\,?}$

False

$-6 \neq 6$

$\sqrt{(-4)(-9)} = \sqrt{36}$

$= 6$

$\sqrt{-4}\sqrt{-9} = \sqrt{4}\sqrt{-1} \cdot \sqrt{9}\sqrt{-1}$

$= (2i)(3i)$

$= (2 \cdot 3)\, i^2$

$= 6(-1)$

$= -6$

Example

Completely simplify each expression

$(-3i)(-7i)$

$(-3i)(-7i) = (-3)(-7)\, i^2$

$= (21)(-1)$

$= -21$

$\sqrt{6}\,\sqrt{-6}$

$\sqrt{6}\sqrt{-6} \xrightarrow{\text{option A}} \sqrt{6}\sqrt{6}\, i = 6i$

$\sqrt{6}\sqrt{-6} \xrightarrow{\text{option B}} \sqrt{-36} = \boxed{\sqrt{36}\sqrt{-1}} = 6i$

$\sqrt{-10}\,\sqrt{-5}$

$\sqrt{-10}\,\sqrt{-5} \xrightarrow{\text{option A}} (\sqrt{10}\, i)(\sqrt{5}\, i) = \sqrt{50}\, i^2$

$= 5\sqrt{2}(-1)$

$= -5\sqrt{2}$

$\sqrt{-10}\,\sqrt{-5} \xrightarrow{\text{option B}} \underbrace{\sqrt{(-10)(-5)}}_{\text{Wrong!}}$

Both numbers are negative; that rule doesn't apply

The Complex Numbers

The Real Numbers

Any number that is represented as a point on the real number line.

The Imaginary Numbers

Any number that is equivalent to a square root of a negative real number

Numbers with a real number part and an imaginary part

Specifically, numbers that can be written in the form $a+bi$ where both a and b are real numbers. This is the standard form in which we write a number that has both a real number part and an imaginary number part.

Example

Perform each operation and write the result in standard complex number form.

$(2+3i)-(7-2i)$

$$(2+3i)-(7-2i) = 2+3i-7+2i$$
$$= -5+5i$$

$(9-2i)(3+4i)$

$$(9-2i)(3+4i) = 27+36i-6i-8i^2$$
$$= 27+30i-8(-1)$$
$$= 35+30i$$

$(4+i)^2$

$$(4+i)^2 = (4+i)(4+i)$$
$$= 16+4i+4i+i^2$$
$$= 16+8i+(-1)$$
$$= 15+8i$$

$(9-i)(10+3i)$

$$(9-i)(10+3i) = 90 + 27i - 10i - 3i^2$$
$$= 90 + 17i - 3(-1)$$
$$= 93 + 17i$$

5 divided by $2-3i$

$$\frac{5}{2-3i} = \frac{5}{2-3i} \cdot \frac{2+3i}{2+3i}$$
$$= \frac{5(2+3i)}{4-9i^2}$$
$$= \frac{5(2+3i)}{4-9(-1)}$$
$$= \frac{10+15i}{13}$$
$$= \frac{10}{13} + \frac{15}{13}i$$

$3+2i$ divided by $3-2i$

$$\frac{3+2i}{3-2i} = \frac{3+2i}{3-2i} \cdot \frac{3+2i}{3+2i}$$
$$= \frac{9+6i+6i+4i^2}{9-4i^2}$$
$$= \frac{9+12i+4(-1)}{9-4(-1)}$$
$$= \frac{5+12i}{13}$$
$$= \frac{5}{13} + \frac{12}{13}i$$

7.6 ; 13-77 odd

MTH 95 – Mr. Simonds' class

$\frac{72}{2i}$

$$\frac{72}{2i} = \frac{72}{2i} \cdot \frac{-2i}{-2i}$$
$$= \frac{-144i}{-4i^2}$$
$$= \frac{-144i}{-4(-1)}$$
$$= -36i$$

$\frac{5-8i}{4i}$

$$\frac{5-8i}{4i} = \frac{5-8i}{4i} \cdot \frac{-4i}{-4i}$$
$$= \frac{-20i+32i^2}{-16i^2}$$
$$= \frac{-20i+32(-1)}{-16(-1)}$$
$$= \frac{-20i-32}{16}$$
$$= -\frac{32}{16} - \frac{20i}{16}$$
$$= -2 - \frac{5}{4}i$$

Example

Complete the following.

$i^1 =$ i	$i^5 = i^4 \cdot i =$ $(1)i = i$	$i^9 =$ i
$i^2 = i^1 \cdot i =$ $i \cdot i = -1$	$i^6 = i^5 \cdot i =$ -1	$i^{10} =$ -1
$i^3 = i^2 \cdot i =$ $-1 \cdot i = -i$	$i^7 = i^6 \cdot i =$ $-i$	$i^{11} =$ $-i$
$i^4 = i^3 \cdot i =$ $(-i)(i) = -(-1) = 1$	$i^8 = i^7 \cdot i =$ 1	$i^{12} =$ 1

Find each of the following.

i^{103}

$$i^{103} = (i^4)^{25} i^3$$
$$= (1)(-i)$$
$$= -i$$

$$4 \overline{)103} = 25 \text{ R}3$$

$-i^{894}$

$$-i^{894} = -(i^4)^{223} i^2$$
$$= -1(-1)$$
$$= 1$$

$$4 \overline{)894} = 223 \text{ R}2$$