Key Concepts: Introduction to rational functions
Simplifying rational expressions
Multiplying and dividing rational expressions

Definition

A function f whose formula can be written in the form $f(x)=\frac{p(x)}{q(x)}$ where p and q are both polynomial functions is called **a rational function**.

Example 1

Graph the function $y=\frac{x-3}{x^2+x-12}$ on your calculator and carefully copy the graph onto Figure 1. State the domain of the function. Finally, write the equation for the function in simplified form.

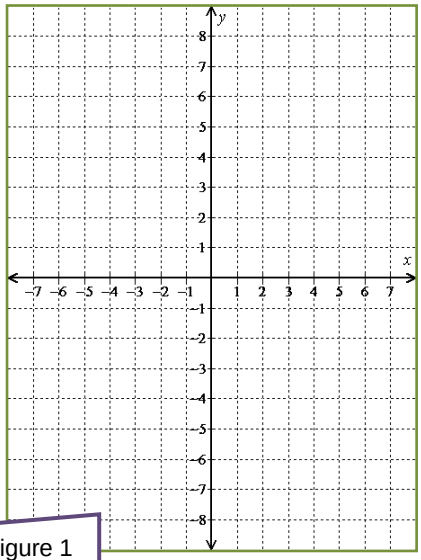

Figure 1

Example 2

Graph the function $y = \dfrac{10}{t^2 + 2t - 8}$ on your calculator and carefully copy the graph onto Figure 2. State the domain of the function. Finally, write the equation for the function in simplified form.

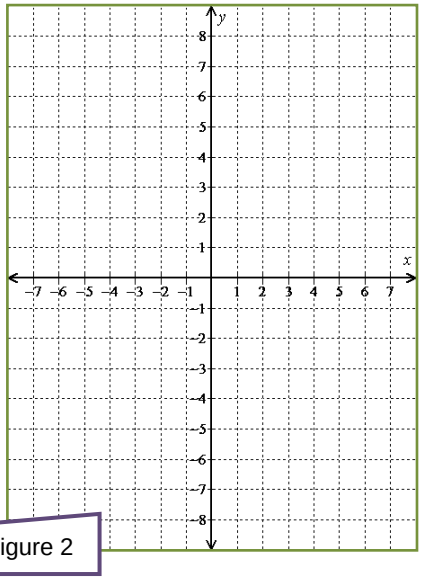

Figure 2

Example 3

Simplify the formula for $g(x) = \dfrac{x(x-5)(2x+1)}{x(2x-1)(x-5)(x+7)}$. Make sure that you state any necessary domain restrictions. State any other numbers that are not in the domain of g.

Example 4

Simplify $\dfrac{x^2-5x+4}{2x^2+5x-12}$. Make sure that you state any necessary domain restrictions.

Recall

To factor $2x^2+5x-12$ we first need to find an integer factor pair of $(2)(-12)$ that adds to 5

Example 4

Simplify each rational expression; make sure that you state any necessary restrictions to the domains.

Simplify $\dfrac{5t+30}{40-10t}$.

Simplify $\dfrac{x^2+7x}{x^2-7x}$.

Simplify $\dfrac{x^4+21x^2}{5x^3+x^7}$.

Example 5

Simplify $\dfrac{t^3-8}{t^2-4t+4}$. State any necessary restrictions on the domain.

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

Example 6

Simplify the formula for $f(y)=\dfrac{y^2-10y-56}{y^2+16}$. What is the domain of f?

Example 7

Simplify the formula for $g(t)=\dfrac{4t^2-9}{2t-3}$. State any necessary restrictions on the domain. What other numbers are not in the domain of g?

Example 8

Simplify $\dfrac{10n+15}{n^2-1}\cdot\dfrac{n+1}{6n+9}$.

Example 9

Simplify $\left(x^2-4\right)\cdot\dfrac{x+2}{4x-8}$.

Example 10

Simplify $\dfrac{4x^4+20x^3+24x^2}{x^2-25}\cdot\dfrac{5x+25}{x^3+27}$.

Example 11

Simplify $\dfrac{x^2-6x-7}{2x+2} \div (x-7)$.

Example 12

Simplify $\dfrac{2x}{x-2} \div \dfrac{x+2}{x} \div \dfrac{7x}{x^2-4}$.