Key Concepts: The Quadratic Formula

Example 1

Use the quadratic formula to solve each equation.

The solutions to $ax^2 + bx + c = 0$ can be found using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$8 - x^2 = 6x$

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$$0 = x^2 + 6x - 8\ ; \quad a = 1,\ b = 6,\ c = -8$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-8)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{36 + 32}}{2}$$
$$x = \frac{-6 \pm \sqrt{68}}{2}$$
$$x = \frac{-6 \pm 2\sqrt{17}}{2}$$
$$x = -3 \pm \sqrt{17}$$

The solutions are $-3 \pm \sqrt{17}$

$2x^2 + \dfrac{x}{3} = 4$

$$2x^2 + \frac{x}{3} = 4$$
$$3\left[2x^2 + \frac{x}{3}\right] = 3[4]$$
$$6x^2 + x = 12$$
$$6x^2 + x - 12 = 0$$
$$a = 6,\ b = 1,\ c = -12$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(6)(-12)}}{2(6)}$$
$$x = \frac{-1 \pm \sqrt{289}}{12}$$
$$x = \frac{-1 \pm 17}{12}$$
$$x = \frac{-1 + 17}{12} \text{ or } x = \frac{-1 - 17}{12}$$
$$x = \frac{4}{3} \text{ or } x = -\frac{3}{2}$$

The solutions are $\frac{4}{3}$ and $-\frac{3}{2}$.

Example 2

State the x-intercepts on each parabola $y=f(x)$ after first solving the equation $f(x)=0$

$y=\frac{x^2}{2}-3x-\frac{3}{4}$ The x-intercepts occur where $y=0$

$$\frac{x^2}{2}-3x-\frac{3}{4}=0$$

$$4\left[\frac{x^2}{2}-3x-\frac{3}{4}\right]=4[\ 0\ ]$$

$$2x^2-12x-3=0$$

$$a=2,\ b=-12,\ c=-3$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(2)(-3)}}{2(2)}$$

$$x=\frac{12\pm\sqrt{144+24}}{4}$$

$$x=\frac{12\pm\sqrt{168}}{4}$$

$$x=\frac{12\pm 2\sqrt{42}}{4}$$

$$x=\frac{\cancel{2}(6\pm\sqrt{42})}{\cancel{2}\cdot 2}$$

$$x=\frac{6\pm\sqrt{42}}{2}$$

The x-intercepts are $\left(\frac{6+\sqrt{42}}{2},0\right)$ and $\left(\frac{6-\sqrt{42}}{2},0\right)$.

$y=x^2+2x+6$

The x-intercepts occur where $y=0$.

$x^2+2x+6=0$; $a=1,\ b=2,\ c=6$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(6)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{-20}}{2} \Leftarrow \text{not real numbers}$$

There is no $-1+\sqrt{5}i$ on the x-axis.

There are no x-intercepts.