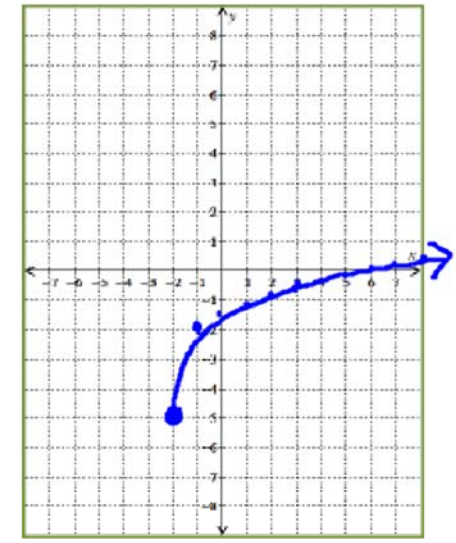
Key Concepts: Graphing Radical Functions
Solving Radical Equations

Example 1

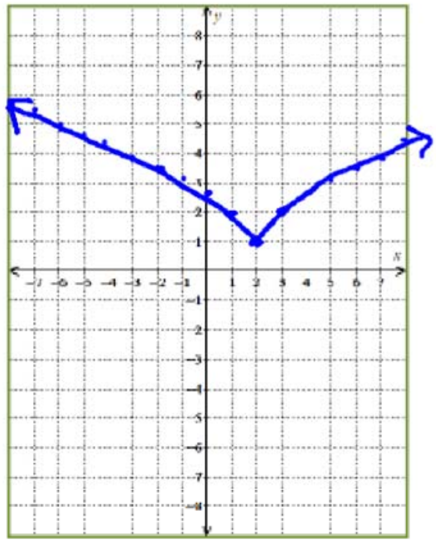
Graph each equation on your calculator and copy the graph onto the provided grid. Then state the domain and range of the function.

Graph $y = 3\sqrt[4]{x+2} - 5$.



The domain is $[-2, \infty)$.
The range is $[-5, \infty)$.

Graph $y = \sqrt[3]{(x-2)^2} + 1$.



The domain is $(-\infty, \infty)$
The range is $[1, \infty)$.
(the squaring limited the range)

Domains and Ranges of Radical Functions

If n is an *odd* positive integer, then both the domain and range of $f(x) = \sqrt[n]{x}$ are $(-\infty, \infty)$.

If n is an *even* positive integer, then both the domain and range of $f(x) = \sqrt[n]{x}$ are $[0, \infty)$.

Example 2

Algebraically determine the domain of each function. Then graph each function on your calculator and copy the graph onto the provided grid. Finally, verify the domain and state the range of the function.

$x-2\geq 0$

MTH 95 – Mr. Simonds' class

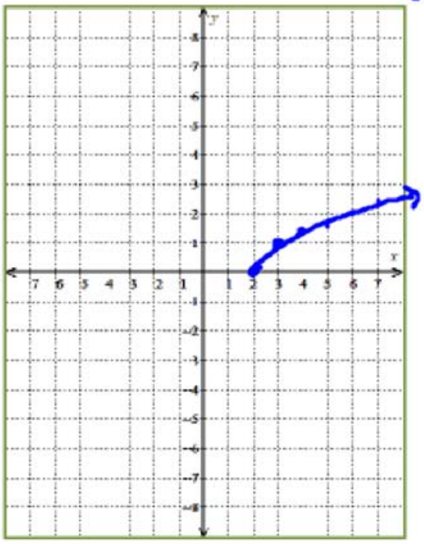
$g(x)=\sqrt{x-2}$

$\sqrt{-7}$

$\sqrt{-1}$

$\sqrt{0}$

even index; the radicand can't be negative



Domain

$x-2\geq 0$

$x\geq 2$

The domain is $[2,\infty)$.

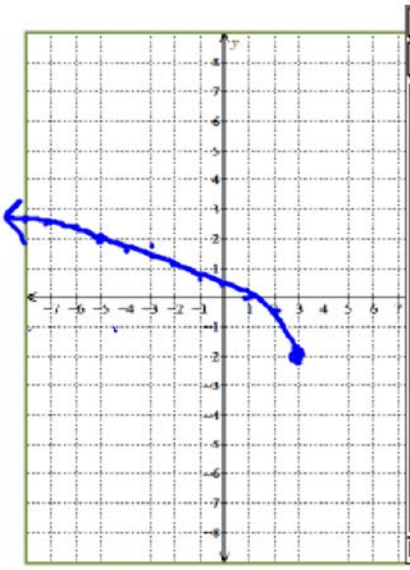
Range

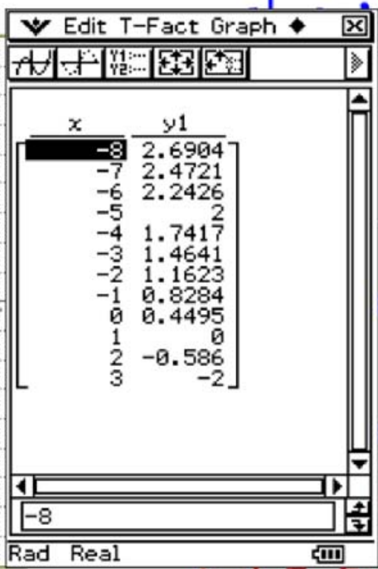
[Even roots are never negative]

The range is $[0,\infty)$.

$f(x)=\sqrt{6-2x}-2$

Even index(2)





x	y1
-8	2.6904
-7	2.4721
-6	2.2426
-5	2
-4	1.7417
-3	1.4641
-2	1.1623
-1	0.8284
0	0.4495
1	0
2	-0.586
3	-2

≥ 0

≥ -6

≤ 3

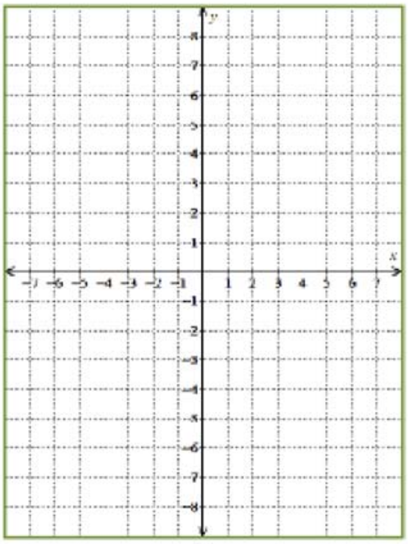
...n is $(-\infty, 3]$

...ts are never negative

$0-2=-2$, $1-2=-1$, $12-2=10$]

The range is $[-2,\infty)$.

$k(x) = \sqrt[3]{2x+6} - 2$

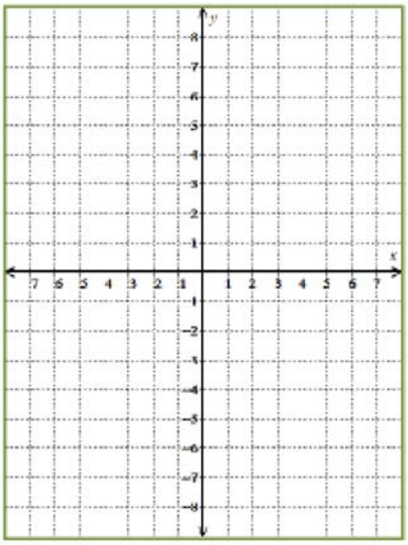


Odd root
no squaring nonsense

o o o

The domain and range are both $(-\infty, \infty)$.

$p(x) = -\sqrt[4]{x+5} + 3$



Even root, so $x+5 \geq 0$

$x \geq -5$

The domain is $[-5, \infty)$

All thought

$$\boxed{p(x) = 3 - \underbrace{\sqrt[4]{x+5}}_{\text{positive or zero}} \leq 3}$$

The range is $(-\infty, 3]$.

$$\sqrt{x} = 2$$
$$(\sqrt{x})^2 = 2^2$$
$$x = 4$$

$$\sqrt{x} = -2$$
$$(\sqrt{x})^2 = (-2)^2$$
$$x = 4$$

Solving equations when the variable occurs in a radicand

1. Isolate the radical expression on one side of the equal sign. If there are two radicals, write them on opposite sides of the equal sign.

2. Raise both sides of the equation to the n^{th} power, where n is the index of the radical expression(s).

3. Solve the resultant equation.

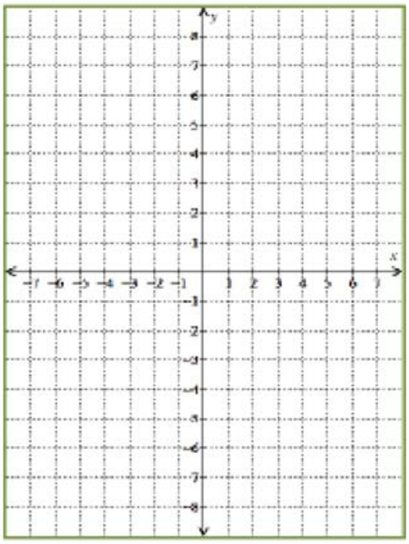
4. You *must* check your solutions. For example, squaring both sides of equation can introduce false solutions.

$-2 = 2$	$4 = 4$ $(-2)^2 = 2^2$
False	True

Example 3

Solve each equation and verify the solution using a graph.

Solve $\sqrt[3]{2-t} = 4$.



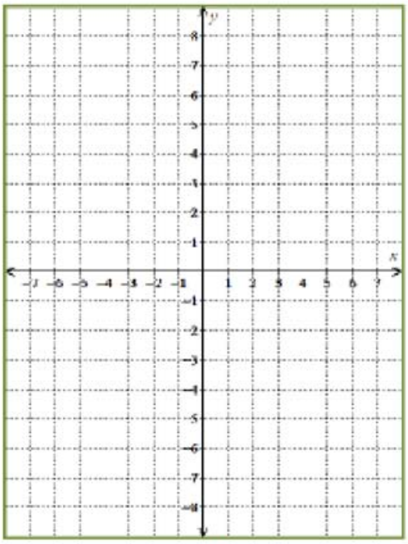
$$\sqrt[3]{2-t} = 4$$
$$\left[\sqrt[3]{2-t}\right]^3 = 4^3$$
$$2 - t = 64$$
$$-62 = t$$

Check

$$\sqrt[3]{2-(-62)} \stackrel{?}{=} 4$$
$$\sqrt[3]{64} = 4$$

The solution is -62.

Solve $\sqrt{x-5}+4=0$.



$$\sqrt{x-5}+4=0$$

$$\sqrt{x-5}=-4$$

$$\left(\sqrt{x-5}\right)^2=(-4)^2$$

$$x-5=16$$

$$x=21$$

There are no solutions to the equation.

Check

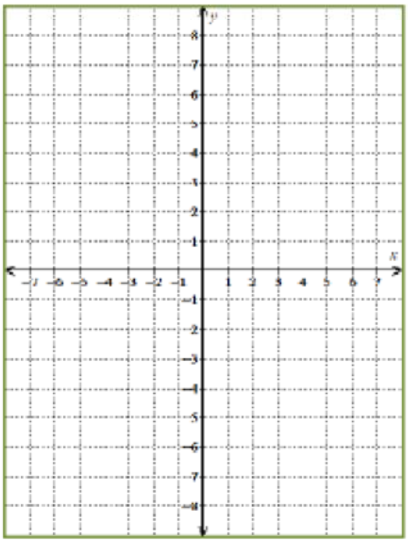
$$\sqrt{21-5}+4\stackrel{?}{=}0$$

$$\sqrt{16}+4\stackrel{?}{=}0$$

$$4+4=0 \leftarrow \text{No!}$$

21 is a poser! When we squared both sides we introduced a false solution.

Solve $2+\sqrt{4x-3}=x$.



$$2+\sqrt{4x-3}=x$$

$$\sqrt{4x-3}=x-2$$

$$\left(\sqrt{4x-3}\right)^2=(x-2)^2$$

$$4x-3=(x-2)(x-2)$$

$$4x-3=x^2-4x+4$$

$$0=x^2-8x+7$$

$$0=(x-7)(x-1)$$

$$x-7=0 \quad \text{or} \quad x-1=0$$

$$x=7 \quad \text{or} \quad x=1$$

Check $x=7$

$$2+\sqrt{4(7)-3}\stackrel{?}{=}7$$

$$2+\sqrt{25}=7 \checkmark$$

Check $x=1$

$$2+\sqrt{4(1)-3}\stackrel{?}{=}1$$

$$2+\sqrt{1}\neq 1$$

The only solution is $\underline{7}$.

Check $x=9$

$\sqrt{9}+\sqrt{7(9)+1} \overset{?}{=} 11$

$3+8=11$ ✓

Check $x=\frac{400}{9}$

$\sqrt{\frac{400}{9}}+\sqrt{7\left(\frac{400}{9}\right)+1} \overset{?}{=} 11$

$\frac{20}{3}+\sqrt{\frac{2809}{9}} \overset{?}{=} 11$

$\frac{20}{3}+\frac{53}{9} \neq 11$ ✓

Solve $\sqrt{x}+\sqrt{7x+1}=11$.

$\sqrt{x}+\sqrt{7x+1}=11$

$-\sqrt{x}$

$\sqrt{7x+1}=11-\sqrt{x}$

$\left(\sqrt{7x+1}\right)^2=\left(11-\sqrt{x}\right)^2$

$7x+1=(11-\sqrt{x})(11-\sqrt{x})$

$7x+1=121-22\sqrt{x}+x$

FOIL

Last

$\sqrt{x}\sqrt{x}=x$

$22\sqrt{x}=120-6x$

$\frac{22\sqrt{x}}{2}=\frac{120-6x}{2}$

$11\sqrt{x}=60-3x$

$\left[11\sqrt{x}\right]^2=\left[60-3x\right]^2$

$121x=3600-360x+9x^2$

9×3600

3 3 36 100

6 6 10 10

2 3 2 3 2 5 2 5

2 2 2 2

3 3 3 3

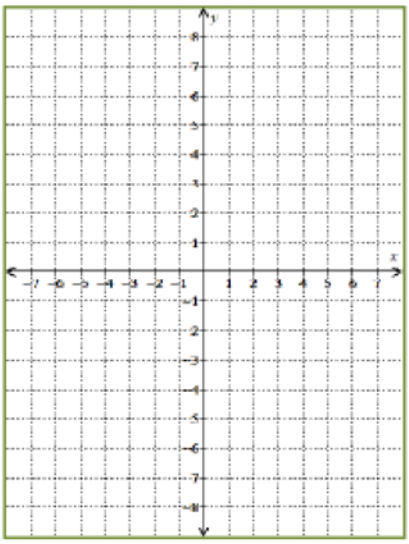
5 5

81

400

$0=9x^2-481x+3600$

we need two factors of (9)(3600) that add to -481.



$0=9x^2-81x-400x+3600$

$0=9x(x-9)-400(x-9)$

$0=(x-9)(9x-400)$

$x-9=0$ or $9x-400=0$

$x=9$ or $x=\frac{400}{9}$

The only solution is 9.