

MTH 95 – Radical Intervention

Section 1 – Simplifying Square Roots

The square root of a number is not considered simplified if it contains a factor that is the perfect square of an integer (other than 1). For example, 4 evenly divides into 12, so we do not consider $\sqrt{12}$ to be simplified. The simplification of this square root is shown below.

$$\begin{aligned}\sqrt{12} &= \sqrt{4 \cdot 3} \\ &= \sqrt{4} \sqrt{3} \\ &= 2\sqrt{3}\end{aligned}$$

Similarly, 4 evenly divides into 48, so we do not consider $\sqrt{48}$ to be simplified. However, $48 = 4 \cdot 12$, and 12 contains another perfect square factor (4 again), so just pulling out $\sqrt{4}$ will not completely simplify $\sqrt{48}$. Two ways of dealing with this are shown below.

$$\begin{aligned}\sqrt{48} &= \sqrt{4 \cdot 12} \\ &= \sqrt{4 \cdot 4 \cdot 3} \\ &= \sqrt{4} \sqrt{4} \sqrt{3} \\ &= (2)(2)\sqrt{3} \\ &= 4\sqrt{3}\end{aligned}$$

$$\begin{aligned}\sqrt{48} &= \sqrt{16 \cdot 3} \\ &= \sqrt{16} \sqrt{3} \\ &= 4\sqrt{3}\end{aligned}$$

As you can see, you can more quickly simplify the square root if you correctly identify the *largest* perfect square that evenly divided into the radicand (the number under the radical sign). However, even if you don't initially find the largest perfect square, you will still end up with the correct simplification if you are vigilant about pulling out additional perfect squares after your first factorization.

Problem 1

List the squares of the integers 2-10.

Problem 2

Shawna was asked to simplify $\sqrt{60}$. She noted that $60 = 6 \cdot 10$ and that neither 6 nor 10 are perfect squares. Shawna concluded from this that $\sqrt{60}$ does not simplify. What's the flaw in Shawna's thinking?

Problem 3

Write each of the following numbers as the product of two factors, one factor that is a perfect square and a second factor that contains no perfect square factors. Do not use your calculator!!

- a. 75 b. 72 c. 700 d. 400 e. 243 f. 112 g. 98 h. 288

Problem 4

Completely simplify each square root showing steps similar to those shown in the examples. Check your answers on your calculator.

a. $\sqrt{75}$ b. $\sqrt{72}$ c. $\sqrt{700}$ d. $\sqrt{400}$ e. $\sqrt{243}$ f. $\sqrt{112}$ g. $\sqrt{98}$ h. $\sqrt{288}$

Section 2 – Simplifying Cube Roots

The cube root of a number is not considered simplified if it contains a factor that is the perfect cube of an integer (other than 1). For example, 27 evenly divides into 54, so we do not consider $\sqrt[3]{54}$ to be simplified. The simplification of this cube root is shown below.

$$\begin{aligned}\sqrt[3]{54} &= \sqrt[3]{27 \cdot 2} \\ &= \sqrt[3]{27} \sqrt[3]{2} \\ &= 3\sqrt[3]{2}\end{aligned}$$

Problem 1

List the cubes of the integers 2-5.

Problem 2

Jimmy was asked to simplify $\sqrt[3]{24}$. Jimmy noted that $24 = 4 \cdot 6$ and that 4 is a perfect square, not a perfect cube. Jimmy concluded from this that $\sqrt[3]{24}$ does not simplify. What's the flaw in Jimmy's thinking?

Problem 3

Write each of the following number as the product of two factors, one factor that is a perfect cube and a second factor that contains no perfect cube factors. Do not use your calculator!!

a. 16 b. 135 c. 48 d. 375 e. 128 f. 432

Problem 4

Completely simplify each cube root showing steps similar to those shown in the example. Check your answers on your calculator.

a. $\sqrt[3]{16}$ b. $\sqrt[3]{135}$ c. $\sqrt[3]{48}$ d. $\sqrt[3]{375}$ e. $\sqrt[3]{128}$ f. $\sqrt[3]{432}$

Section 3 – Rationalizing Monomial Denominators that Contain Square Roots

Many math instructors do not consider a number simplified if it contains a square root in its denominator. When a denominator is a monomial that contains a square root, we can eliminate the square root in the denominator by multiplying both the numerator and denominator of the fraction by the square root factor in the denominator. Of course, this leaves us with a square root in the numerator but even fussy mathematicians are not bothered by this.

Several examples of this simplification process are shown below.

$$\begin{aligned}\frac{2}{\sqrt{5}} &= \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{2\sqrt{5}}{5}\end{aligned}$$

$$\begin{aligned}\frac{6}{\sqrt{3}} &= \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{6\sqrt{3}}{3} \\ &= 2\sqrt{3}\end{aligned}$$

Make sure that you reduce the fraction *formed by the factors that are not underneath the square root sign.*

$$\begin{aligned}\frac{4}{3\sqrt{6}} &= \frac{4}{3\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{4\sqrt{6}}{3(6)} \\ &= \frac{4\sqrt{6}}{18} \\ &= \frac{2\sqrt{6}}{9}\end{aligned}$$

There is **no reason** to multiply the numerator and denominator by 3; 3 is not the irrational factor!

Make sure that you reduce the fraction *formed by the factors that are not underneath the square root sign.*

$$\begin{aligned}\frac{7}{\sqrt{12}} &= \frac{7}{\sqrt{4 \cdot 3}} \\ &= \frac{7}{2\sqrt{3}} \\ &= \frac{7}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{7\sqrt{3}}{2(3)} \\ &= \frac{7\sqrt{3}}{6}\end{aligned}$$

You can keep the numbers as small as possible if you first simplify the square root in the denominator.

There is **no reason** to multiply the numerator and denominator by 2; 2 is not the irrational factor!

Problem 1

Rationalize each denominator and simplify the result. **Check** your answers on your calculator.

a. $\frac{7}{\sqrt{7}}$ b. $\frac{6}{5\sqrt{3}}$ c. $\frac{7}{2\sqrt{5}}$ d. $\frac{3}{\sqrt{50}}$ e. $\frac{6}{\sqrt{2}}$ f. $\frac{4}{\sqrt{18}}$

Section 4 – Rationalizing Binomial Denominators that Contain Square Roots

The trick here is to multiply the numerator and denominator by the conjugate of the denominator.

The expressions $a + b$ and $a - b$ are called conjugates and they have the special property that when you FOIL them the O and I add to zero so that in the end all you are left with is $a^2 - b^2$.

Two examples of this simplification process are shown below

$$\begin{aligned}\frac{2}{2 - \sqrt{6}} &= \frac{2}{2 - \sqrt{6}} \cdot \frac{2 + \sqrt{6}}{2 + \sqrt{6}} \\ &= \frac{2(2 + \sqrt{6})}{4 - 6} \\ &= \frac{2(2 + \sqrt{6})}{-2} \\ &= -1 \cdot (2 + \sqrt{6}) \\ &= -2 - \sqrt{6}\end{aligned}$$

Make sure that you reduce the fraction ***when there are common factors in the numerator and denominator.***

$$\begin{aligned}\frac{5 + \sqrt{10}}{5 - \sqrt{10}} &= \frac{5 + \sqrt{10}}{5 - \sqrt{10}} \cdot \frac{5 + \sqrt{10}}{5 + \sqrt{10}} \\ &= \frac{25 + 5\sqrt{10} + 5\sqrt{10} + 10}{25 - 10} \\ &= \frac{35 + 10\sqrt{10}}{15} \\ &= \frac{5(7 + 2\sqrt{10})}{5 \cdot 3} \\ &= \frac{7 + 2\sqrt{10}}{3}\end{aligned}$$

Remember, we are multiplying both the numerator and the denominator by the ***conjugate of the denominator.***

Make sure that you reduce the fraction ***when there are common factors in the numerator and denominator.***

Problem 1

Rationalize each denominator and simplify the result. **Check** your answers on your calculator. Please note that on your calculator you will need to use the "simplify" and possible "expand" commands to see the full simplification.

$$\text{a. } \frac{4}{2 + \sqrt{7}} \quad \text{b. } \frac{12}{\sqrt{6} - 10} \quad \text{c. } \frac{1 - \sqrt{5}}{1 + \sqrt{5}} \quad \text{d. } \frac{\sqrt{2}}{\sqrt{2} + 2} \quad \text{e. } \frac{3 + \sqrt{6}}{3 - \sqrt{6}}$$

Section 5 – Combining Radicals

Like radical expressions can be added or subtracted in the same way that like terms can be added or subtracted. Often, though, you need to simplify the terms before you can even determine if you have any like radicals. When confronting two or more radical terms, you want to, in a sense, treat each term as its "own problem;" once you have each term simplified, you can then combine any like radicals. Several examples of this process are shown below.

$$\begin{aligned}\sqrt{32} + \sqrt{18} &= \sqrt{16 \cdot 2} + \sqrt{9 \cdot 2} \\ &= 4\sqrt{2} + 3\sqrt{2} \\ &= 7\sqrt{2}\end{aligned}$$

$$\begin{aligned}5\sqrt{24} - \frac{12}{\sqrt{6}} &= 5\sqrt{4 \cdot 6} - \frac{12}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \\ &= 5(2)\sqrt{6} - \frac{12\sqrt{6}}{6} \\ &= 10\sqrt{6} - 2\sqrt{6} \\ &= 8\sqrt{6}\end{aligned}$$

$$\begin{aligned}\sqrt{3} + 3\sqrt{75} + \frac{6}{2 + \sqrt{3}} &= \sqrt{3} + 3\sqrt{25 \cdot 3} + \frac{6}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} \\ &= \sqrt{3} + 3(5)\sqrt{3} + \frac{6(2 - \sqrt{3})}{4 - 3} \\ &= \sqrt{3} + 15\sqrt{3} + 12 - 6\sqrt{3} \\ &= 12 + 10\sqrt{3}\end{aligned}$$

Problem 1

Completely simplify each expression. **Check** your answers on your calculator. Please note that on your calculator you may need to use the "simplify" and possible "expand" commands to see the full simplification.

a. $\sqrt{64} + \sqrt{36}$

b. $\sqrt{45} - 7\sqrt{20}$

c. $\frac{1}{\sqrt{2}} + 2\sqrt{72}$

d. $\frac{3}{\sqrt{3}} + 2\sqrt{27} - 4\sqrt{18}$

e. $3\sqrt{68} + \frac{16}{\sqrt{17}-1}$

f. $\frac{8-4\sqrt{2}}{2\sqrt{2}} + \frac{1}{1+\sqrt{2}}$

Section 6 – Rational Exponents

By definition, $x^{m/n} = \sqrt[n]{x^m}$. When using this rule, you need to pay special attention to the presence or lack of grouping symbols. For example,

$$8x^{1/3} = 8\sqrt[3]{x} \text{ where-as } (8x)^{1/3} = \sqrt[3]{8x} = \sqrt[3]{8}\sqrt[3]{x} = 2\sqrt[3]{x}$$

Also, when numbers get involved it can be simpler if you apply the radical part first. For example:

$$\begin{aligned} 81^{3/2} &= (\sqrt{81})^3 \\ &= 9^3 \\ &= 729 \end{aligned}$$

Now you may not be thrilled having to multiply out 9^3 by hand, but think about the alternative (cubing out 81 and then figuring out the square root of that freaking number).

Problem 1

Write each expression in radical form and simplify the results. **Check** your answers on your calculator.

a. $8^{4/3}$

b. $25y^{1/2}$

c. $7w^{-6/5}$

d. $(28y)^{1/2}$

e. $(2z)^{-1/2}$

f. $16^{-5/2}$

g. $(9x^{20})^{3/2}$

h. $125^{-4/3}$

i. $(64w^{12})^{-1/3}$

j. $(12t^8)^{-3/2}$