Key Concepts: Solving equations and inequalities graphically
Connecting absolute value inequalities to graphical inequalities

Example 1

In figures 1-3, function f and the line $y=3$ are shown three times. $f(x) = |x+2|$

a. On Figure 1, mark the points on f with y-coordinates of 3. Mark the corresponding points on the x-axis and state the solution set to the equation $f(x)=3$.

from the graph $f(x)=3$ at -5 and 1

$|x+2| = 3$

$x+2 = 3$ or $x+2 = -3$

$x = 1$ or $x = -5$

The solution set is $\{-5, 1\}$

b. On Figure 2, mark the points on f with y-coordinates greater than 3. Mark the corresponding intervals on the x-axis and state the solution set to the inequality $f(x)>3$.

$f(x) > 3$

$|x+2| > 3$

$x+2 > 3$ or $x+2 < -3$

$x > 1$ or $x < -5$

The solution set is $(-\infty, -5) \cup (1, \infty)$

c. On Figure 3, mark the points on f with y-coordinates less than or equal to 3. Mark the corresponding interval on the x-axis and state the solution set to the inequality $f(x) \le 3$

$f(x) \le 3$

$|x+2| \le 3$

$-3 \le x+2 \le 3$

$-5 \le x \le 1$

The solution set is $[-5, 1]$

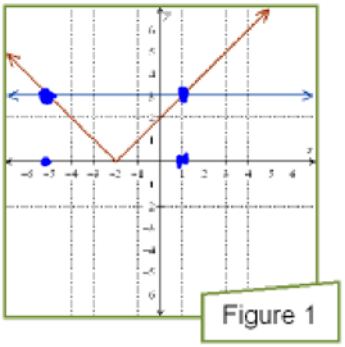
Figure 1

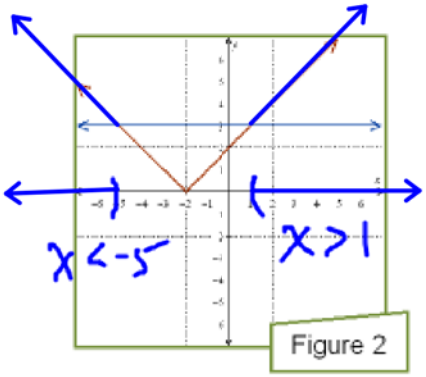

Figure 2

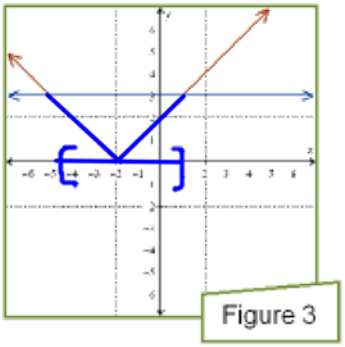
Figure 3

The end points of the solution set to an inequality always make the two sides equal

Example 2

The formula for f is $f(x)=|x+2|$. Use Figure 4 to solve the inequalities $|x+2|<-4$ and $|x+2|\geq -4$

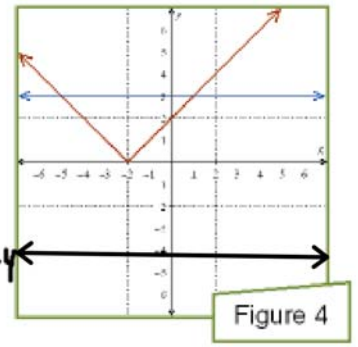

Figure 4

All of the y-coordinates on $y=|x+2|$ are greater than -4.

The solution set to $|x+2|<-4$ is $\emptyset$.

The solution set to $|x+2|\geq -4$ is $\mathbb{R}$.

Example 3

The graph $y=2|x-2|-5$ is shown in Figure 5.

a. Use the graph to determine the solution set to $2|x-2|-5\geq 1$.

b. Verify your solution algebraically.

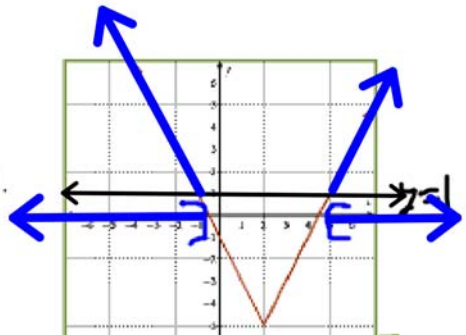

Figure 5

$2|x-2|-5\geq 1$

$2|x-2|\geq 6$

$|x-2|\geq 3$

$x-2\geq 3$ or $x-2\leq -3$

$x\geq 5$ or $x\leq -1$

The solution set is $(-\infty,-1]\cup[5,\infty)$

Example 4

The graph $y = 2|x-2|-5$ is shown in Figure 6.

a. Use the graph to determine the solution set to $2|x-2|-5<3$.

b. Verify your solution algebraically.

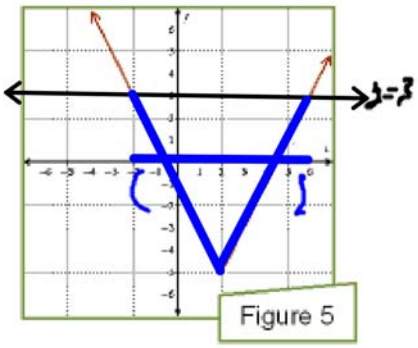

Figure 5

$2|x-2|-5<3$

$2|x-2|<8$

$|x-2|<4$

$-4<x-2<4$

$-2<x<6$

The solution set is $(-2,6)$.

Example 5

Graph $y = 5-|x+1|$ on your graphing calculator and carefully copy the graph onto Figure 7. Then use the graph to solve the inequality $5-|x+1|\geq 0$.

$5-|x+1|\geq 0$

$-|x+1|\geq -5$

$|x+1|\leq 5$

$-5\leq x+1\leq 5$

$-6\leq x\leq 4$

The solution set is $[-6,4]$.

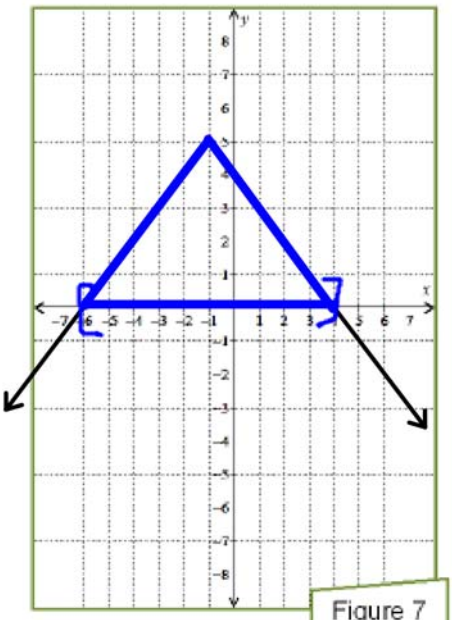

Figure 7

Example 6

Graph $y = 3|x| - 6$ on your graphing calculator and carefully copy the graph onto Figure 8. Then use the graph to solve the inequality $3|x| - 6 > -3$.

$3|x| - 6 > -3$

$3|x| > 3$

$|x| > 1$

$x > 1$ or $x < -1$

The solution set is $(-\infty, -1) \cup (1, \infty)$.

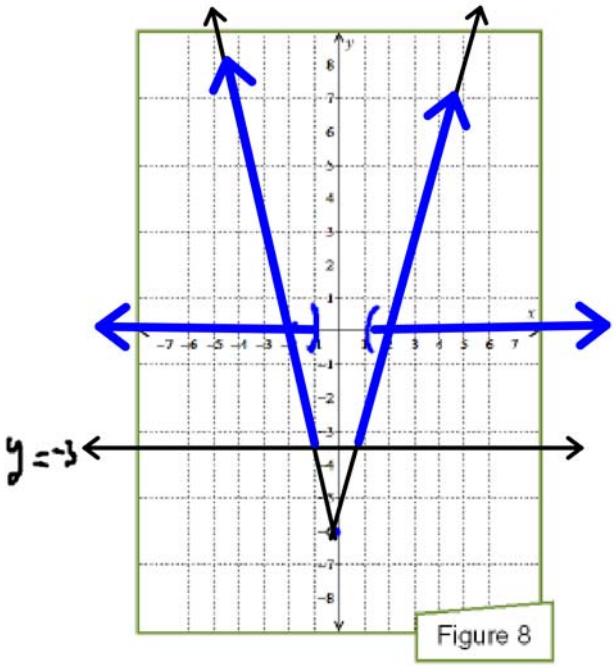

Figure 8

Example 7

Graph $y = 5 - \frac{2}{3}|x|$ on your graphing calculator and carefully copy the graph onto Figure 9. Then use the graph to solve the inequality $5 - \frac{2}{3}|x| \geq 1$.

$5 - \frac{2}{3}|x| \geq 1$

$-\frac{2}{3}|x| \geq -4$

$-\frac{3}{2}\left(-\frac{2}{3}|x|\right) \leq -\frac{3}{2}(-4)$

$|x| \leq 6$

$-6 \leq x \leq 6$

The solution set is $[-6, 6]$.

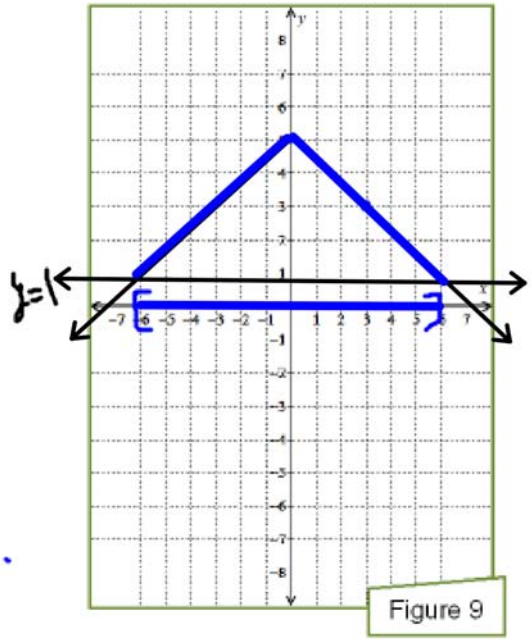

Figure 9