

Key Concepts: Compound Inequalities
Absolute Value Equations and Inequalities

Intersections and unions

Suppose that **A** and **B** are two sets of numbers.

The **intersection** of **A** and **B** is the set of all numbers that are in *both A and B*. This set is symbolized as $A \cap B$.

The **union** of **A** and **B** is the set of all numbers that are in *either A and/or B*. This set is symbolized as $A \cup B$.

Example 1

Suppose that **A** is the set $\{1, 2, 3, 4, 5, 6\}$ and **B** is the set $\{1, 3, 5, 7, 9, 11\}$.

a. What are $A \cup B$ and $A \cap B$?

b. What are $A \cup \emptyset$, $A \cap \emptyset$, $A \cup \mathbb{R}$, and $A \cap \mathbb{R}$?

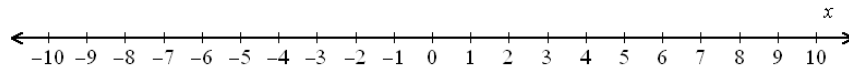
Recall

\emptyset is a symbol used for the **empty set** - the set with no elements. This set is also denoted by the symbol $\{\}$.

\mathbb{R} is a symbol used for the **set of all real numbers**. This set is also denoted (using interval notation) as $(-\infty, \infty)$.

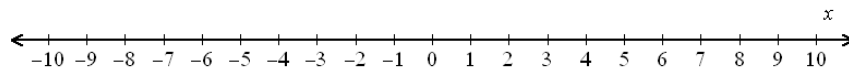
Example 2

Graph the set $(-\infty, -2) \cup (4, \infty)$ and write a compound inequality whose solution set matches the graph.



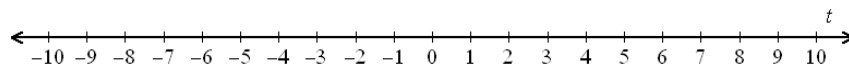
Example 3

Graph the solution set to the compound inequality $x \geq 6$ or $x < 0$. State the solution set using interval notation.



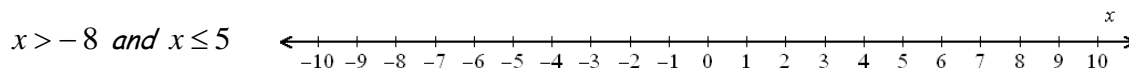
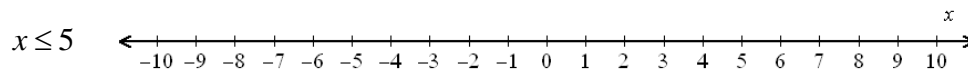
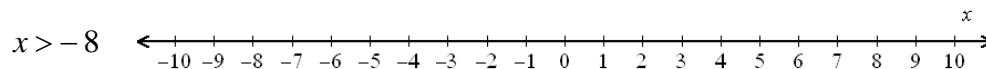
Example 4

Graph the solution set to the compound inequality $3t + 1 \geq 10$ or $5 - 3t \geq 14$. State the solution set using interval notation.



Example 5

Graph the solution set to each inequality and state the solution set to the compound inequality using interval notation.

**Solutions to “and” inequalities**

The solution set to an “and” inequality between two linear inequalities is *always* one of three things: the empty set, the set of all real numbers, or an interval *between* two numbers (which may or may not include the endpoints).

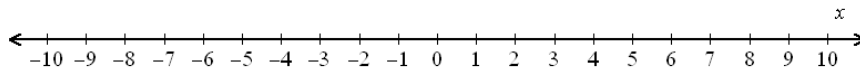
For this reason, “and” inequalities are often written as “between” inequalities. For example, we could write $x > 3$ and $x < 7$ as $3 < x < 7$.

When writing inequalities where the variable is written between two numbers, the two inequality signs *must always open in the same direction*.

$$a < x > b \quad \leftarrow \text{WRONG!!} \rightarrow \quad a > x < b$$

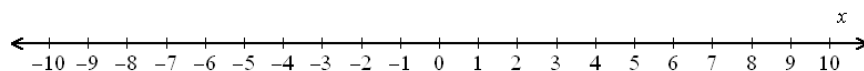
Example 6

Graph the interval $[-9, -2)$ and then write an “and” inequality and a “between” inequality whose solution sets graph to that interval.

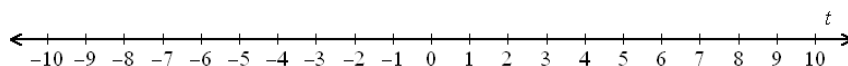


Your turn ... work each of the following problems with one or two partners.

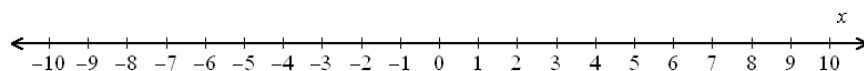
1. Graph the solution set to the compound inequality $x > -4$ *and* $x < 8$. State the solution set using interval notation.



2. Find and graph the solution set to the compound inequality $2 - 4t \geq 10$ *or* $4 + 2t > 10$. State the solution set using interval notation.



3. Find and graph the solution set to the compound inequality $-5 \leq 4 - x \leq 4$. State the solution set using interval notation. Also, write a compound inequality using the word *and* that has the same solution set.



4. Writing $3 \leq x \leq 7$ is bad, m'kay? Rewrite that expression as a compound inequality using the word *and* and discuss what's bad about the situation.



Absolute value equations and inequalities

Suppose that k is a positive real number.

The equation $|y| = k$ is equivalent to the compound equation $y = k$ or $y = -k$.

The inequality $|y| < k$ is equivalent to the compound inequality $-k < y < k$.

The inequality $|y| > k$ is equivalent to the compound inequality $y > k$ or $y < -k$.

Example 7

Rewrite each equation or inequality as a compound equation or inequality.

$$|2x + 1| \geq 5$$

$$|t + 5| < 9$$

$$|3 - 5x| \leq 9$$

$$|2 - t| + 3 = 8$$

$$-2|x| - 5 < 7$$

Example 8

Find the solution set to each equation or inequality after first rewriting the inequality as a compound inequality. *Begin by completely isolating the absolute value expression.*

$$45 - 9|x - 4| = 0$$

$$|3 - 2x| + 1 < 6$$

$$2|x + 6| - 6 \geq -3$$

Example 9

Suppose that $f(x) = 2x + 1$ and $g(x) = 3 - x$. Find the solution set to each of the following inequalities.

$$-2|g(x)| > 4$$

$$|f(x)| < 0$$

$$|g(x)| \geq -5$$

$$-2|f(x+3)| + 3 \leq 11$$

Your turn ... work each of the following problems with one or two partners.

1. Find the solution set to $|x| > 7$. Write the solution set using interval notation.
2. Find the solution set to $|x| - 10 < -4$. Write the solution set using interval notation.
3. Find the solution set to $|x - 10| < -4$.
4. Find the solution set to $|f(t)| \geq 8$ where $f(t) = 3 - 5t$. Write the solution set using interval notation.
5. Find the solution set to $|f(4 - x)| = 4$ where $f(x) = 2x - 7$.
6. Find the solution set to $-3|g(x)| + 2 < -10$ where $g(x) = 2x + 4$.