MTH 95 – Absolute Intervention

Section 1 – Writing an absolute value equation or inequality in its equivalent compound form (part 1)

If N is a ***positive*** real number then each of the following is true.

- The equation $|ax+b|=N$ is equivalent to the compound equality:

$$ax+b=-N \text{ or } ax+b=N$$

- The inequality $|ax+b|<N$ is equivalent to the compound inequality:

$$-N<ax+b<N$$

- The inequality $|ax+b|>N$ is equivalent to the compound equality:

$$ax+b<-N \text{ or } ax+b>N$$

When solving an absolute equation or inequality, the first step is to completely isolate the absolute value expression. Assuming that the resultant number on the other side of the equal sign is positive, the next step to rewrite the equation or inequality in its equivalent compound form. This process is shown in the following three examples.

$|x-5|-7=12$
$|x-5|=19$ (add 7 to both sides of the equation to isolate the absolute value expression)
$x-5=-19$ or $x-5=19$ (write the equivalent compound equation)

$3+2|5x-1|\le 7$
$2|5x-1|\le 4$ (subtract 3 from both sides of the inequality)
$|5x-1|\le 2$ (divided both sides of the inequality by 2)
$-2\le 5x-1\le 2$ (write the equivalent compound inequality)

$-6|4-x|<-18$
$|4-x|>3$ (divide both sides of the inequality by -6; note that the inequality sign reverses direction)
$4-x<-3$ or $4-x>3$ (write the equivalent compound inequality)

Problem 1

Write the equivalent compound equation or inequality for each of the following. ***Do not go any further in the problem solving process***.

a. $|2x+5| \le 7$ b. $|3+6t| = 10$ c. $|x-8| < 9$ d. $|3-4z| > 8$

e. $|5-x| \ge 12$ f. $|3y+14| = 17$ g. $5 \le |m+7|$ h. $16 > |x|$

Problem 2

Write the equivalent compound equation or inequality for each of the following after first completely isolating the absolute value expression. ***Do not go any further in the problem solving process***.

a. $2+|x-6| \ge 10$ b. $3|4x| = 18$ c. $4|2w+6|+2 < 14$

d. $3-|3x-1| = 1$ e. $-6|5n+2| > -30$ f. $4-2|5-x| \le -10$

Problem 3

Go ahead and completely solve each of the equations and inequalities in problems 1 and 2. State the ***solution <u>set</u>*** for each equation or inequality.

Section 2 – Exceptional Absolute Value Equations and Inequalities

The absolute value of a number is never negative. Consequently, the solution sets for both $|x| = -9$ and $|3+2x| < -14$ are empty. On the other hand, every absolute value is greater than any negative real number, so the solutions set for $|4-5x| \ge -16$ is all real numbers.

Problem 1

The solution set for each of the following is either the empty set or all real numbers. Decide the solution set for each.

a. $|x+5| \le -7$ b. $|6m| = -100$ c. $|w+8| > -19$ d. $|3+4z| \ge -1$

e. $|5-x| \ge 12$ f. $|3y+14| = 17$ g. $5 \le |m+7|$ h. $16 > |x|$

Problem 2

Find the solution set for each of the following.

a. $3+|x-4| < 1$ b. $5-2|2x+1| > 5$ c. $3-6|4m+4| \le 27$

d. $|3+2y|+5 = 5$ e. $-6|5n+2| \le 30$ f. $7-2|3x+1| \ge 7$