Functions

Recall that a set of ordered pairs, $\{(x, y)\}$, defines y as a function of x if and only if no two ordered pairs in the set have the same x-coordinate. If the set is a function named f and the ordered pair (a, b) is in the set, then we write $f(a) = b$ and we say that the function value at a is b.

Example 1

Fill in the missing information about a function called g.

Point on g	Function Notation	What we say
$(2, -3)$	$g(2) = -3$	The function value at 2 is -3
$(7, 12)$	$g(7) = 12$	The function value at 7 is 12
$(8, 22)$	$g(8) = 22$	The function value at 8 is 22.
$(0, 51)$	$g(0) = 51$	The function value at 0 is 51
$(1, -14)$	$g(1) = -14$	The function value at 1 is -14.

Example 2

Translate each statement into the appropriate symbols.

The value of f at 9 is 62

$$f(9) = 62$$

f at x is less than 20

$$f(x) < 20$$

k defines y as a function of x (y is a function of x)

$y = K(x)$

12 is the value of k at -4 $12 = K(-4)$

$K(-4) = 12$

g of t is equal to 17

$g(t) = 17$

Example 3
Answer each question about the function w. function value at x is y

What ordered pair is in w if the value of w at 8 is 22?

$(8, 22)$ $w(8) = 22$

What ordered pair is in w if 94 is the function value at 55?

$(55, 94)$

What ordered pair is in w if the function value at -19 is 7?

$(-19, 7)$

What ordered pair is in w if 82 is the value of w at -3?

$w(7) = 10$

$(-3, 82)$

What function value do you know if the ordered pair $(7,10)$ is in w?

I know that the function value at 7 is 10.

Example 4
Answer each of the following questions about the function f whose graph is shown in Figure 1.

What is the function value at 4?

The function value is -1.

Write an equation that states the function value at 4.

$f(4) = -1$

What is the function value at -3?

2

Write an equation that states the function value at -3.

$f(-3) = 2$

Same question, different wording

At what values of x is the function value equal to 0? These values are called <u>the zeros of f.</u>

The zeros of f are -5 and 3

What are the solutions to the equation $f(x) = 0$?

The solutions are -5 and 3.

At what values of x is the function value equal to 0?

The function value equals 0 at -5 and 3.

What are the solutions to the equation $f(x) = 2$?

The solutions are -3 and 1

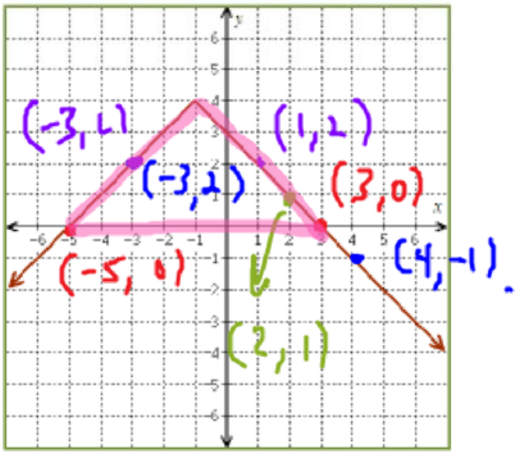


Figure 1: $y = f(x)$

1

What is the value of f at 2?

The value is 1.

Between what two values of x is the function value always positive?

We're looking for points above the x-axis.
The function value is positive between -5 and 3.

Example 5
Fill in each blank about the function k in Figure 2.

$k(\underline{\ 4\ }) = 3$ $\quad k(3) = \underline{\ 2.5\ }$ $\quad k(-1) = \underline{\ 1/2\ }$ $\quad k(\underline{\ -4\ }) = -1$

$y = 4$

The value of k is 4 at $\underline{\ 6\ }$. The function value at 4 is $\underline{\ 3\ }$.

$\hookrightarrow x$

The function value at 0 is $\underline{\ 1\ }$. The zero of k is $\underline{\ -2\ }$.

$x = 0$

where does $y = 0$.

The slope of the line is $\underline{\ \frac{1}{2}\ }$. The y-intercept of the line is $\underline{(0, 1)}$.

$\frac{2}{4}$ $\quad m = \frac{1}{2}$ $\quad b = 1$

Example 6
What is the formula for the function k in Figure 2? Use that formula to find the function value at -1, 0, 3, and 4.

Slope-intercept form $y = mx + b$

$$y = \frac{1}{2}x + 1$$

but we have new name for y!

The function formula is

$$K(x) = \frac{1}{2}x + 1$$

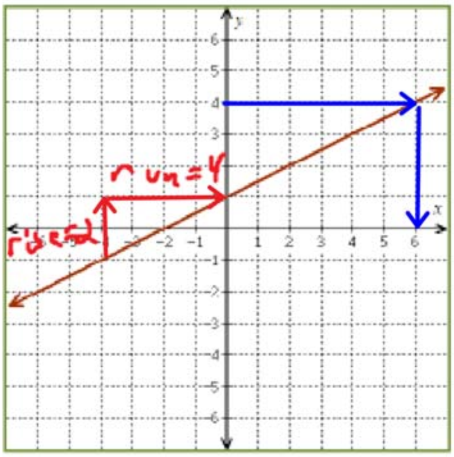


Figure2: $y = k(x)$

$$K(-1) = \frac{1}{2}(-1) + 1 = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$K(0) = \frac{1}{2}(0) + 1 = 1$$

$$K(3) = \frac{1}{2}(3) + 1 = 5/2$$

$$K(4) = \frac{1}{2}(4) + 1 = 3$$

Example 7

Determine each of the following about the function $g(x) = x^2 - 4x - 5$.

What is the value of g at -1?

$$g(-1) = (-1)^2 - 4(-1) - 5$$
$$= 1 + 4 - 5$$
$$= 0$$

What is the value of $g(0)$.

$$g(0) = 0^2 - 4(0) - 5$$
$$= -5$$

What is the function value at 0?

The function value at 0 is -5.

What are the zeros of g?

where does $y = 0$

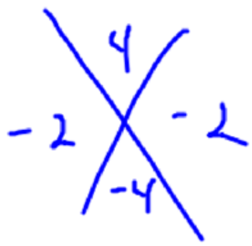
$$g(x) = 0$$
$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$

$x - 5 = 0$ or $x + 1 = 0$

$x = 5$ or $x = -1$

The zeros are 5 and -1.

What are the solutions to the equation $g(x) = -9$?



$$g(x) = -9$$
$$x^2 - 4x - 5 = -9$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)(x-2) = 0$$
$$x - 2 = 0$$
$$x = 2$$

The only solution is 2.

$$f(\boxed{x+4}) = (\boxed{x+4})^2 + 6$$

Example 8

Find $f(3)$, $f(-5)$, $f(x)$, $f(x+4)$, and $f(x)+4$ if $f(t)=t^2+6$.

$$f(3) = 3^2+6 = 15$$

$$f(x) = x^2+6$$

$$f(-5) = -5^2+6 = 25+6 = 31$$

$$f(x)+4 = (x^2+6)+4 = x^2+10$$

$$f(x+4) = (x+4)^2+6 = (x+4)(x+4)+6 = x^2+8x+16+6 = x^2+8x+22$$

Example 9

Find $g(-2)$, $g(17)$, $g(t)$, $g(t)-7$, and $g(t-7)$ if $g(x)=4-3x$.

$$g(-2) = 4-3(-2) = 4+6 = 10$$

$$g(17) = 4-3(17) = 4-51 = -47$$

$$g(t) = 4-3t$$

$$g(t)-7 = 4-3t-7 = -3t-3$$

$$g(t-7) = 4-3(t-7) = 4-3t+21 = -3t+25$$

$f(\square) = (\square)^2 + (\square)$

Example 10

Find $f(3)$, $f(-1)$, $f(x)+9$, and $f(x+9)$ if $f(x)=x^2+x$.

$f(3) = 3^2+3$
$= 12$

$f(-1) = (-1)^2 + (-1)$
$= 1-1$
$= 0$

$f(x)+9 = x^2+x+9$

$f(x)+9 = (x^2+x)+9$
$= x^2+x+9$

$f(x+9) = (x+9)^2 + (x+9)$
$= x^2+18x+81+x+9$
$= x^2+19x+90$

$(x+9)(x+9)$

Example 11

Find $w(0)$, $w(19)$, $w(t-2)$, and $w(t)-2$ if $w(t)=3t-20$.

$w(0) = 3(0)-20$
$= -20$

$w(19) = 3(19)-20$
$= 37$

$w(t-2) = 3(t-2)-20$
$= 3t-6-20$
$= 3t-26$

$w(t)-2 = (3t-20)-2$
$= 3t-22$

Check
$j(-2) = -3(-2) - 5$
$= 6 - 5$
$= 1$ ✓

$j(1) = -3(1) - 5$
$= -8$ ✓

Therefore $j(x) = -3x - 5$

Example 12

Find the formula for the linear function j if you know that $j(-2)=1$ and $j(1)=-8$. (Use x as the independent variable.)

x	y
-2	1
1	-8

run = 3, rise = -9

$x_1 = -2 \quad y_1 = 1$
$x_2 = 1 \quad y_2 = -8$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-1}{1-(-2)} = -3$$

Option A
point-slope
$y - y_1 = m(x - x_1)$
$y - 1 = -3(x-(-2))$
$y - 1 = -3x - 6$
$y = -3x - 5$

Option B
slope-intercept
$y = mx + b$
we know $m = -3$
$y = -3x + b$
plug in $x = -2, y = 1$

$1 = -3(-2) + b$
$1 = 6 + b$
$-5 = b$
$y = -3x - 5$

Example 13

Find the value of h at 7 where h is the linear function with $h(9)=19$ and $h(-1)=4$. (Use x as the independent variable.)

$x_1 = 9 \quad y_1 = 19$
$x_2 = -1 \quad y_2 = 4$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-19}{-1-9} = \frac{-15}{-10} = \frac{3}{2}$$

$y = mx + b$
$y = \frac{3}{2}x + b$
$x = -1, y = 4$
$4 = \frac{3}{2}(-1) + b$
$4 + \frac{3}{2} = b$
$\frac{11}{2} = b$

Therefore $h(x) = \frac{3}{2}x + \frac{11}{2}$
and
$h(7) = \frac{3}{2}(7) + \frac{11}{2}$
$= \frac{21+11}{2}$
$= \frac{32}{2}$
$= 16$

$f(t)=12$

Example 14

Determine where the value of the linear function f is 12 if $f(3)=1$ and $f(-14)=18$. (Use t as the independent variable.)

$x_1=3 \quad y_1=1$

$x_2=-14 \quad y_2=18$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{18-1}{-14-3}=\frac{17}{-17}=-1$$

$y=mx+b$

$m=-1,\ x=3,\ y=1$

$1=(-1)(3)+b$

$4=b$

$y=-1\cdot x+4$

Therefore, $f(t)=-t+4$.

$12=-t+4$

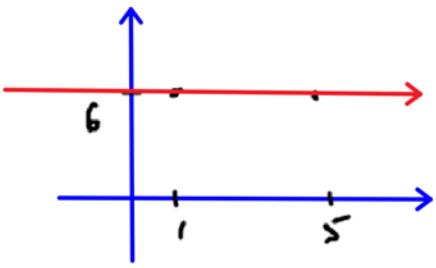
$t=-8$

The function value is 12 at -8.

$f(-8)=12$

Example 15

Find the value of g at 11 where g is the linear function with $g(5)=6$ and $g(1)=6$. (Use t as the independent variable.)



$g(t)=6 \Leftarrow$ This is called a constant function.

So $g(11)=6$

Example 16

Dieter made 50 slingshots that he is selling from a booth he set up on his front lawn. The amount of profit Dieter makes if he sells x slingshots is given by the function $P(x) = 3x - 18$. Answer each of the following questions about Dieter's profit function.

What is the contextual domain of the function?

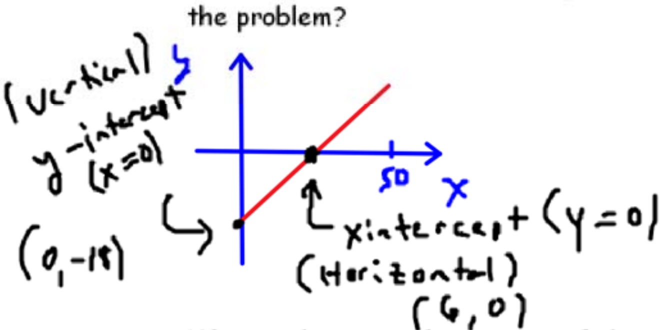
The domain is $\{0, 1, 2, \ldots, 48, 49, 50\}$

What is the function value at 15 and what does it tell you in the context of the problem?

$$P(15) = 3(15) - 18$$
$$= 27$$

If Dieter sells 15 slingshots, he will make $27 in profit.

What is the horizontal intercept of the profit function and what does it tell you in the context of the problem?



The horizontal-intercept is (6, 0) which tells us that Dieter will break even if he sells 6 slingshots.

What is the vertical intercept of the profit function and what does it tell you in the context of the problem?

The vertical intercept is (0, -18) which tells us that Dieter's upfront costs were $18.

What is the slope of the profit function and what does it tell you in the context of the problem?

The slope is 3 which tells us that Dieter charges $3 per slingshot.

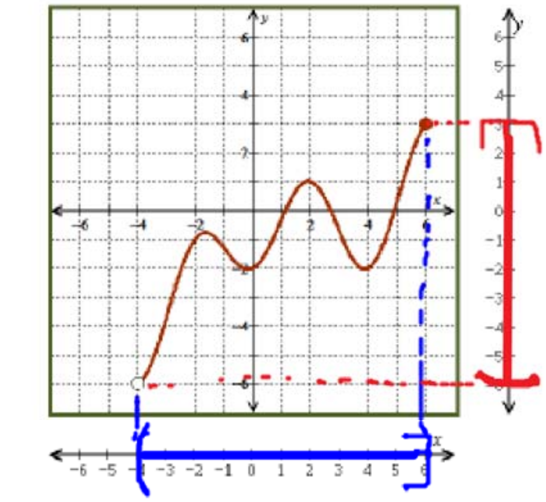
Domain and Range

If the set of ordered pairs, $\{(x, y)\}$ is a function, then the set of all of the x-coordinates is called the domain of the function and the set of all of the y-coordinates is called the range of the function.

Example 17

any set of points

Consider the set of points on each of the following curves. For each relation, indicate whether or not the relation is also a function. Also state the domain and range of each relation using interval notation.

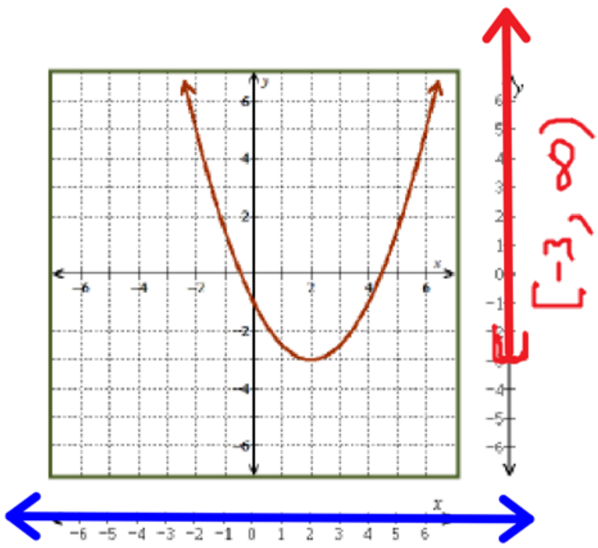


Function? yes

Domain $(-4, 6]$

Range $(-6, 3]$

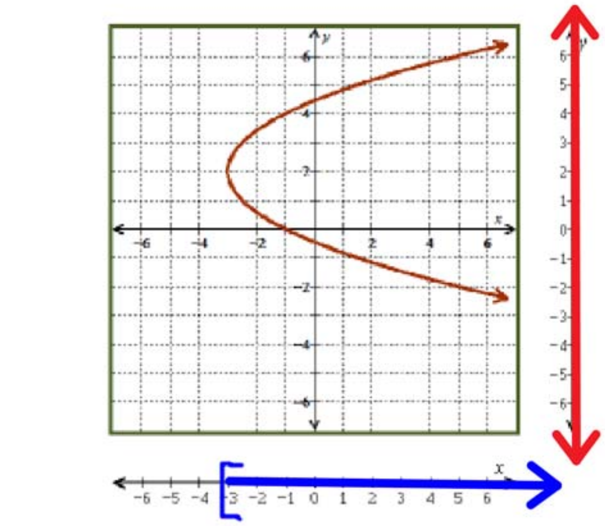
$\mathbb{R}$



Function? yes

Domain $(-\infty, \infty)$

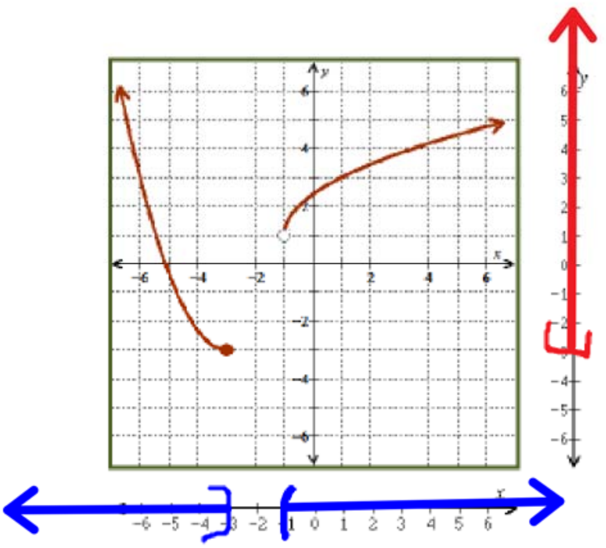
Range $[-3, \infty)$



Function? no

Domain $[-3, \infty)$

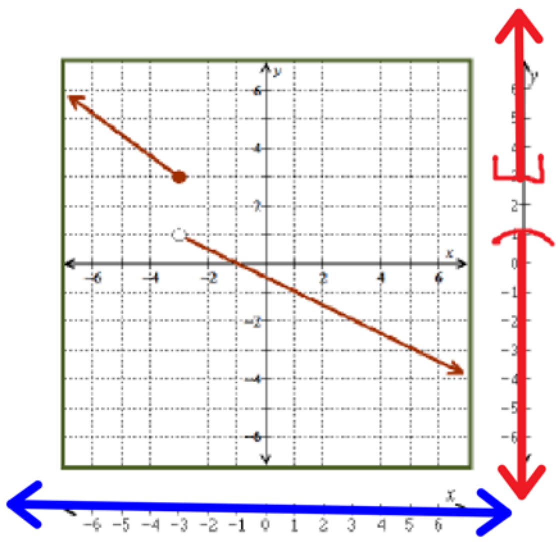
Range $(-\infty, \infty)$



Function? yes

Domain $(-\infty, -3] \cup (-1, \infty)$

Range $[-3, \infty)$



Function? yes

Domain $(-\infty, \infty)$

Range $(-\infty, 1) \cup [3, \infty)$