MTH 65 – Winter Term 2009
Final Exam – Given March 18
Calculator allowed portion

Name Key

1. Use the quadratic formula to solve the equation $4x^2 - 4x - 9 = 0$. Make sure that you completely simplify your solutions and that you clearly state your solutions.

$4x^2 - 4x - 9 = 0$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-9)}}{2(4)}$

$x = \dfrac{4 \pm \sqrt{160}}{8}$

$x = \dfrac{4 \pm \sqrt{16 \cdot 10}}{8}$

$x = \dfrac{4 \pm 4\sqrt{10}}{8}$

$x = \dfrac{1 \pm \sqrt{10}}{2}$

The solutions are $\dfrac{1+\sqrt{10}}{2}$ and $\dfrac{1-\sqrt{10}}{2}$

2. Solve each equation using whatever method you desire. Make sure that you completely simplify your solutions and that you clearly state your solutions.

a. Solve $x^2 - 6x + 9 = 0$.

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x - 3 = 0$ or $x - 3 = 0$

umm... $x = 3$!

The solution is 3.

b. Solve $(t+3)^2 = t(t+7)$.

$(t+3)^2 = t(t+7)$

$(t+3)(t+3) = t^2 + 7t$

$t^2 + 6t + 9 = t^2 + 7t$

$9 = t$

The solution is 9.

3. Solve each system of equations using the specified methods. Make sure that you clearly state your solutions.

   a. Use the addition (elimination) method to solve system $\begin{cases} 2x+5y=17 \\ 6x-7y=-37 \end{cases}$.

$\begin{cases} 2x+5y=17 \\ 6x-7y=-37 \end{cases} \Rightarrow \begin{cases} -3(2x+5y)=-3(17) \\ 6x-7y=-37 \end{cases}$

$\Rightarrow \begin{cases} -6x-15y=-51 \\ 6x-7y=-37 \end{cases}$

$-22y=-88$

$y=4$

Backsub 4 for y in $2x+5y=17$

$2x+5(4)=17$

$2x=-3$

$x=-3/2$

The solution is $(-\frac{3}{2}, 4)$.

   b. Use the substitution method to solve the system $\begin{cases} -3x-12y=24 \\ x=-4y-8 \end{cases}$

Substitute $-4y-8$ for x in the first equation

$-3x-12y=24$

$-3(-4y-8)-12y=24$

$12y+24-12y=24$

$0=0$ ← I'd agree with that!

Every point on the line $-3x-12y=24$ is a solution to this system!

4. Jennifer bungee jumps off of a bridge exactly 500 ft above a river. The length of the cord strapped around Jennifer's feet is 80 feet (before stretching). While she is in free fall the height (ft) above the ground of Jennifer's feet t seconds after she jumps is given by the function $h(t) = 500 - 16t^2$.

   a. How high above the river are Jennifer's feet when the cord begins to stretch (assuming she is directly below the point at which she jumped when the cord begins to stretch).

Jennifer falls 80 feet before the cord begins to stretch, so her feet are 420 ft above the river when the cord first stretches.

   b. How long (to the nearest 100th of a second) is Jennifer in free fall? That is, how long does it take after Jennifer jumps before the cord begins to stretch?

$h(t) = 420$

$500 - 16t^2 = 420$

$80 = 16t^2$

$5 = t^2$

$\pm\sqrt{5} = t$

Since the cord doesn't stretch before she jumps,

$t = \sqrt{5} \approx 2.24$

∴ It took about 2.24 seconds for the cord to begin to stretch.

5. There are two different numbers that when squared are 6 more than the original number. Write an equation that models this situation, solve the equation, and state the two numbers.

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

The numbers are 3 and −2.

5. Completely factor each expression showing work in the manner illustrated in class.

   a. Factor $3x^4 - 30x^3 + 72x^2$.

$3x^4 - 30x^3 + 72x^2 = 3x^2(x^2 - 10x + 24)$

$= 3x^2(x-6)(x-4)$

   b. Factor $20t^2 - 28t - 3$.

$20t^2 - 28t - 3 = 20t^2 - 30t + 2t - 3$

$= 10t(2t-3) + 1(2t-3)$

$= (2t-3)(10t+1)$

6. Gumballs cost 16 cents apiece and gummy worms cost 19 cents apiece. Yoshi bought a total of 60 of these gum objects and spent a total \$10.29. Let x represent the number of gumballs Yoshi bought and y represent the number of gummy worms Yoshi bought. Write a system of two equations in terms of x and y that models this situation, solve the system, and state the contextual solution.

$\begin{cases} x + y = 60 \\ .16x + .19y = 10.29 \end{cases} \Rightarrow \begin{cases} -16(x+y) = -16(60) \\ 100(.16x + .19y) = 100(10.29) \end{cases}$

$\Rightarrow \begin{cases} -16x - 16y = -960 \\ 16x + 19y = 1029 \end{cases}$

$3y = 69$

$y = 23$

$x + 23 = 60 \Rightarrow x = 37$

Yoshi bought 37 gumballs and 23 gummy worms.

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1. Write each requested bit of information in the provided blank. **Do all of your work on the provided scratch paper**. These questions will be marked right or wrong.

a. Expand and simplify $(x-3)(x+7)$. a. $x^2+4x-21$

b. Expand and simplify $2(x+1)(x-4)$. b. $2x^2-6x-8$

c. Expand and simplify $(5w+3)(5w-3)$. c. $25w^2-9$

d. Expand and simplify $(1+2y)^2$. d. $1+4y+4y^2$

e. Completely factor $x^2-4x-21$. e. $(x-7)(x+3)$

f. Completely factor $8x^3-1$. f. $(2x-1)(4x^2+2x+1)$

g. Completely factor $4x^2+9y^2$. g. Prime!

h. Completely factor $w^2-14w+49$. h. $(w-7)^2$

i. Completely factor $x^2+4xy+4y^2$. i. $(x+2y)^2$

2. Write each requested bit of information in the provided blank. **Do all of your work on the provided scratch paper.** These questions will be marked right or wrong. Make sure that none of your answers contain negative exponents.

a. Completely simplify $\frac{x^2 x^7}{x^{14}}$. a. $\frac{1}{x^5}$

b. Completely simplify $\frac{y^7}{2y^{-3}}$. b. $\frac{y^{10}}{2}$

c. Completely simplify $\frac{\left(3x^{-4}\right)^{-2}}{x^8}$. c. $\frac{1}{9}$

d. Completely simplify $\sqrt{294}$. d. $7\sqrt{6}$

e. Completely simplify $\sqrt{288}$. e. $12\sqrt{2}$

f. State all solutions to $(x+1)^2 = 4$. f. 1 and -3

g. State all solutions to $x^2 + 9 = 0$. g. no real solutions

h. What is the linear coefficient in $4x + 7$? h. 4

i. What is the degree of the term $6x^2 y^9 z$? i. 12

3. Consider the function f shown in Figure 1.

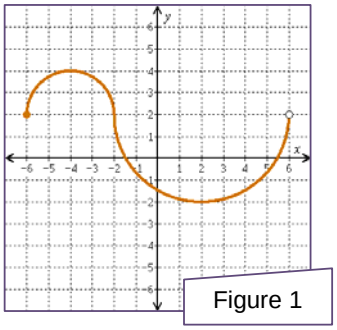

Figure 1

a. State the domain and range of f.

The domain is $[-6, 6)$.

The range is $[-2, 4]$.

b. State (using an equation) the value of $f(-2)$.

$f(-2) = 2$.

4. Find the value of $g(-3)$ is $g(t) = t^2 - 9$. Make sure that you show the work the way it was illustrated and discussed in class,

$$g(-3) = (-3)^2 - 9$$
$$= 9 - 9$$
$$= 0$$

5. Completely simplify each expression showing steps organized in a manner consistent with that illustrated and discussed during class.

a. Simplify $\left(\frac{4x^{-1}y^{2}}{x^{5}y^{-8}}\right)^{-1}$.

$$\left(\frac{4x^{-1}y^{2}}{x^{5}y^{-8}}\right)^{-1} = \frac{4^{-1}xy^{-2}}{x^{-5}y^{8}}$$
$$= \frac{xx^{5}}{4y^{2}y^{8}}$$
$$= \frac{x^{6}}{4y^{10}}$$

b. Simplify $-4^2 + 4^{-2}$.

$$-4^2 + 4^{-2} = -16 + \frac{1}{16}$$
$$= -15\frac{15}{16}$$
$$= -\frac{255}{16}$$

6. State (in the provided blanks) the vertex and intercepts of the parabola $y = x^2 - 6x - 16$. Then fill into the table 9 ordered pairs that clearly illustrate the symmetry of the parabola. Finally, graph the parabola onto the provided axes making sure that you give appropriate consideration to the placement of the axes and the scale to use on each axis.

Vertex: $(3, -25)$ y-intercept: $(0, -16)$

x-intercepts: $(8, 0)$ and $(-2, 0)$

x	y
-1	-9
0	-16
1	-21
2	-24
3	-25
4	-24
5	-21
6	-16
7	-9
8	0
9	11