MTH 65 – Winter Term 2008
Final Exam – Given March 19
No Calculator Portion

Name Key

1. Write each requested bit of information in the provided blank. Please do all of your scratch work on the provided scratch paper and write only you final answer in the blank. (2 pts each)

Please note: None of the correct answers contain negative exponents.

a. Expand and simplify $(2x+1)(x-7)$ a. $2x^2-13x-7$

b. Expand and simplify $(m+9)^2$ b. $m^2+18m+81$

c. Completely simplify $\frac{x^{-2}}{x^{-8}}$ c. x^6

d. Completely simplify $(3b^4)^2$ d. $9b^8$

e. Factor, completely, $t^2-5t-50$ e. $(t-10)(t+5)$

f. Factor, completely, $16x^2-25$ f. $(4x-5)(4x+5)$

g. Factor, completely, m^3+8 g. $(m+2)(m^2-2m+4)$

h. Completely simplify $-8x^{-3}$ h. $-\frac{8}{x^3}$

i. Expand and simplify $(x-3)(x^2+x+1)$ i. x^3-2x^2-2x-3

2. Use the zero property to solve the equation $(x-3)(x+4)=8$. Make sure that each of your solutions is completely simplified. (5 points)

$(x-3)(x+4)=8$

$x^2-3x+4x-12=8$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x+5=0$ OR $x-4=0$

$x=-5$ OR $x=4$

The solutions are -5 and 4.

3. Use the quadratic formula to solve the equation $\frac{x^2}{4}-\frac{x}{2}-1=0$. Make sure that each of your solutions is completely simplified. (5 points)

$\frac{x^2}{4}-\frac{x}{2}-1=0$

$4\left(\frac{x^2}{4}-\frac{x}{2}-1\right)=4(0)$

$x^2-2x-4=0$

$a=1,\ b=-2,\ c=-4$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$

$=\frac{2\pm\sqrt{20}}{2}$

$x=\frac{2\pm\sqrt{4\cdot5}}{2}$

$=\frac{2\pm2\sqrt{5}}{2}$

$=\frac{2}{2}\pm\frac{2\sqrt{5}}{2}$

$=1\pm\sqrt{5}$

The solutions are $1+\sqrt{5}$ and $1-\sqrt{5}$.

4. Consider the parabola with equation $y = x^2 - 8x - 3$. (8 points)

   a. What is the vertex of the parabola?

$a = 1, b = -8$

$x = -\frac{b}{2a} = \frac{-8}{-2} = 4$

when $x = 4$,

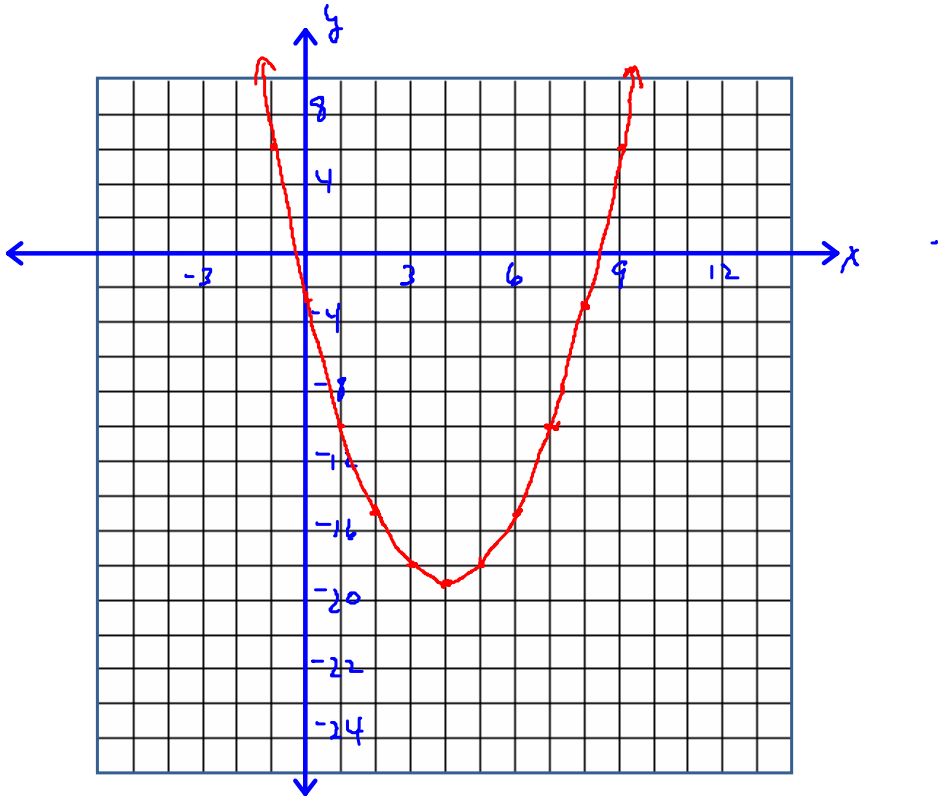
$y = 4^2 - 8(4) - 3$

$= -19$

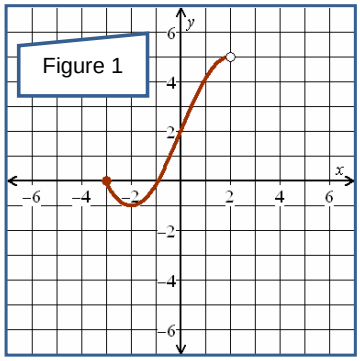
The vertex is $(4, -19)$

   b. Draw a complete graph of the parabola on the grid below. When choosing your axes, make sure that you make a choice that allows you to clearly show the symmetry of the parabola and that also allows you to graph all of the intercepts of the parabola.

x	y
-1	6
0	-3
1	-10
2	-15
3	-18
4	-19
5	-18
6	-15
7	-10
8	-3
9	6



5. Consider the function h shown in Figure 1. (8 points)



a. What is the value of $h(-2)$?

$h(-2) = -1$

b. For what values of x does $h(x) = 2$?

0

c. What are the domain and range of h?

Domain: $[-3, 2)$ Range: $[-1, 5]$

6. Find $f(-3)$ if $f(x) = \sqrt{x^2 + 16}$. (3 points)

$$f(-3) = \sqrt{(-3)^2 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

7. Find the x-intercepts and y-intercept of the parabola $y = x^2 + 16$. (4 points)

The y-intercept is $(0, 16)$

There are no x-intercepts because $x^2 + 16$ never equals 0.

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1. Write each requested bit of information in the provided blank. Please do all of your scratch work on the provided scratch paper and write only you final answer in the blank.

Please note: In each of the factor problems you need to make sure that the expression is *completely* factored.

a. Factor $6 + x - 12x^2$ — a. $(3-4x)(2+3x)$

b. Factor $4a^2 - 36a + 72$ — b. $4(a-3)(a-6)$

c. Factor $4x^2 + 81y^2$ — c. $4x^2+81y^2$ is prime

d. Factor $16x^2z^4 - 8xy^3z^2 + y^6$ — d. $(4xz^2 - y^3)^2$

e. What is k if $\left(x^k\right)^7 = x^{84}$? — e. 12

f. What is k if $b^k b^{20} = b^{37}$? — f. 17

g. What is k if $-8^k = -1$? — g. 0

h. Completely simplify $\dfrac{\left(x^2y^{-3}\right)^{-2}}{xy^{-3}}$ — h. y^9/x^5

i. Completely simplify $\dfrac{(-3x)^4}{-3x^4}$ — i. -27

2. A pop tart is tossed into the air from the top of a building on planet Xenon. The height above the ground (ft) of the pop tart t seconds after it is tossed is given by the function $h(t) = -10t^2 + 50t + 240$. Answer each of the following questions about this pop tart and make sure that you show all of the work that leads to your conclusions.

   a. How high above the ground was the pop tart when it was tossed?

$h(0) = 240$

The tart was tossed from a height of 240 ft.

   b. What was the maximum height reached by the pop tart?

The vertex of the parabola occurs at $t = \frac{-b}{2a} = \frac{-50}{-20} = 2.5$

$h(2.5) = 302.5$

So the maximum height reached by the pastry was 302.5 ft.

   c. How many seconds did it take for the pop tart to fall to the ground?

$h(t) = 0 \Rightarrow -10t^2 + 50t + 240 = 0$

$\Rightarrow -10(t^2 - 5t - 24) = 0$

$\Rightarrow -10(t-8)(t+3) = 0$

$\Rightarrow t - 8 = 0$ or $t + 3 = 0$

$\Rightarrow t = 8$ or $t = -3$.

Since the tasty pastry didn't fall to the ground before it was tossed, it took 8 seconds for the pop tart to fall to the ground.

3. Use the elimination method to solve the system of equations $\begin{cases} 3x - 4y = 20 \\ 5x - 2y = 24 \end{cases}$.

$\begin{cases} 3x - 4y = 20 \\ 5y - 2y = 24 \end{cases} \Rightarrow \begin{cases} 3x - 4y = 20 \\ -2(5y - 2y) = -2(24) \end{cases}$

$\Rightarrow \begin{cases} 3x - 4y = 20 \\ -10x + 4y = -48 \end{cases}$

$-7x = -28$

$\Rightarrow x = 4$

Replacing x with 4 is the first equation

$3(4) - 4y = 20 \Rightarrow -4y = 8 \Rightarrow y = -2$

The solution to the system of equations is $(4, -2)$.

4. Use the substitution method to solve the system of equations $\begin{cases} -3x + 12y = 19 \\ x - 4y = -3 \end{cases}$.

Solving the second equation for x we get $x = 4y - 3$

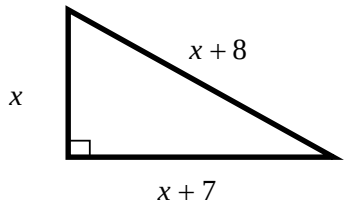
Replacing x with $4y - 3$ in the first equation gives us

$-3(4y - 3) + 12y = 19 \Rightarrow -12y + 9 + 12y = 19$

$\Rightarrow 9 = 19$ ← NOT GONNA HAPPEN!

This system of equations has no solutions.

5. Find a quadratic equation that models the Pythagorean Theorem for the triangle below and solve that equation to determine the value of x.



$x^2 + (x+7)^2 = (x+8)^2$

$x^2 + (x+7)(x+7) = (x+8)(x+8)$

$x^2 + x^2 + 7x + 7x + 49 = x^2 + 8x + 8x + 64$

$2x^2 + 14x + 49 = x^2 + 16x + 64$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x - 5 = 0$ OR $x + 3 = 0$

$x = 5$ OR $x = -3$

Since length can't be negative,

$x = 5$