MTH 65 – Practice Final – Version 1 Solution Key

1. Solving the second equation for x we have $x = 18 + 2y$. Substituting $18 + 2y$ for x in the first equation gives us:

$$\begin{aligned} 2(18 + 2y) - 3y &= 29 \\ 36 + 4y - 3y &= 29 \\ 36 + y &= 29 \\ y &= -7 \end{aligned}$$

When $y = -7$, $x = 18 + 2y = 4$. The solution to the system is $(4, -7)$.

2. $\begin{cases} 3x - 4y = 1 \\ 4x + 6y = -27 \end{cases}$ is equivalent to $\begin{cases} 3(3x - 4y) = 3(1) \\ 2(4x + 6y) = 2(-27) \end{cases}$ which simplifies to

$\begin{cases} 9x - 12y = 3 \\ 8x + 12y = -54 \end{cases}$. Adding the respective sides of the new system gives us:

$$17x = -51 \;\Rightarrow\; x = -3$$

Substitution -3 for x in the original second equation we have:

$$4(-3) + 6y = -27 \;\Rightarrow\; 6y = -15 \;\Rightarrow\; y = \frac{-15}{6} = -\frac{5}{2}$$

The solutions to the system is $\left(-3, -\frac{5}{2}\right)$.

3. Note: Show me as much work as you need to show to get the correct expansion. If you can do it entirely in your head (correctly!), that's fine. e.g., you would get full credit for the writing $(w-9)^2 = w^2 - 18w + 81$.

$$\boxed{\begin{aligned} (2x + y)(3x - y) &= 6x^2 - 2xy + 3xy - y^2 \\ &= 6x^2 + xy - y^2 \end{aligned}}$$

$$\boxed{\begin{aligned} (w-9)^2 &= (w-9)(w-9) \\ &= w^2 - 9w - 9w + 81 \\ &= w^2 - 18w + 81 \end{aligned}}$$

$$\boxed{\begin{aligned} (2x+3)(4x^2 - 6x + 9) &= 8x^3 - 12x^2 + 18x + 12x^2 - 18x + 27 \\ &= 8x^3 + 27 \end{aligned}}$$

4. Note: Show me as much work as you need to show to get the correct simplification. If you can do it entirely in your head (correctly!), that's fine. e.g., you would get full credit for the writing $\dfrac{-2^{-2}a^2}{a^{-4}b^{-12}} = -\dfrac{a^6 b^{12}}{4}$.

$$\boxed{\begin{aligned}\frac{35x^7 y}{7x^2 y^8} &= \frac{35}{7}\cdot\frac{x^7}{x^2}\cdot\frac{y}{y^8}\\ &= \frac{5x^5}{y^7}\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{-2^{-2}a^2}{a^{-4}b^{-12}} &= \frac{-2^{-2}}{1}\cdot\frac{a^2}{a^{-4}}\cdot\frac{1}{b^{-12}}\\ &= \frac{-1}{2^2}\cdot\frac{a^2 a^4}{1}\cdot\frac{b^{12}}{1}\\ &= -\frac{a^6 b^{12}}{4}\end{aligned}}$$

$$\boxed{\begin{aligned}10x^{-1} &= \frac{10}{1}\cdot\frac{x^{-1}}{1}\\ &= \frac{10}{1}\cdot\frac{1}{x}\\ &= \frac{10}{x}\end{aligned}}$$

$$\boxed{\begin{aligned}\left(-4a^8 b^{-4}\right)^2 &= (-4)^2\left(a^8\right)^2\left(b^{-4}\right)^2\\ &= 16a^{16}b^{-8}\\ &= \frac{16a^{16}}{b^8}\end{aligned}}$$

$$\boxed{\begin{aligned}\left(\frac{2x^3 y z^{-1}}{3x^0 y^8 z^{-1}}\right)^{-3} &= \frac{2^{-3}x^{-9}y^{-3}z^3}{3^{-3}\cdot 1\cdot y^{-24}z^3}\\ &= \frac{3^3}{2^3}\cdot\frac{1}{x^9}\cdot\frac{y^{24}}{y^3}\cdot\frac{z^3}{z^3}\\ &= \frac{27y^{21}}{8x^9}\end{aligned}}$$

5. a. $-21{,}500{,}000{,}000 = -2.15\times10^{10}$ b. $0.000000000000091\times10^{-14}$

6. Write each number in standard notation.

a. $-7.2\times10^{-1} = -0.72$ b. $8.88\times10^{11} = 888{,}000{,}000{,}000$

7. Note: Show me as much work as you need to show to get the correct simplification. If you can do it entirely in your head (correctly!), that's fine. e.g., you would get full credit for the writing $\left(2.2\times10^{-8}\right)\left(5\times10^3\right) = 1.1\times10^{-4}$.

$$\boxed{\begin{aligned}\left(2.2\times10^{-8}\right)\left(5\times10^3\right) &= 11\times10^{-5}\\ &= 1.1\times10^1\times10^{-5}\\ &= 1.1\times10^{-4}\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{4.2\times10^{-5}}{8.4\times10^{-9}} &= 0.50\times10^4\\ &= 5.0\times10^{-1}\times10^4\\ &= 5.0\times10^3\end{aligned}}$$

8. $$\begin{aligned}f(-7) &= -(-7)^2 - 8(-7) + 50\\ &= -49 + 56 + 50\\ &= 57\end{aligned}$$

9.

$$\boxed{\begin{aligned}8x^2+6x-35&=8x^2+20x-14x-35\\&=4x(2x+5)-7(2x+5)\\&=(4x-7)(2x+5)\end{aligned}}$$

$$\boxed{\begin{aligned}16w^4-1&=(4w^2-1)(4w^2+1)\\&=(2w-1)(2w+1)(4w^2+1)\end{aligned}}$$

$$\boxed{4x^2+y^2 \text{ is prime}}$$

$$\boxed{12y^2+4xy+16x^2=4(3y^2+xy+4x^2)}$$

$$\boxed{\begin{aligned}x^2+9xy-90y^2&=x^2+15xy-6xy-90y^2\\&=x(x+15y)-6y(x+15y)\\&=(x-6y)(x+15y)\end{aligned}}$$

10.

$$w=\frac{-8\pm\sqrt{8^2-4(1)(-8)}}{2(1)}$$

$$w=\frac{-8\pm\sqrt{96}}{2}$$

$$w=\frac{-8\pm\sqrt{16\cdot 6}}{2}$$

$$w=\frac{-8\pm\sqrt{16}\sqrt{6}}{2}$$

$$w=\frac{-8\pm 4\sqrt{6}}{2}$$

$$w=-4\pm 2\sqrt{6}$$

The solutions to the equation are:

$-4\pm 2\sqrt{6}$

11.

$$x^2-10x+21=0$$

$$(x-3)(x-7)=0$$

$x-3=0$ or $x-7=0$

$x=3$ or $x=7$

The solutions to the equation are 3 and 7.

12.

$$(3t+9)^2-180=0$$

$$(3t+9)^2=180$$

$$3t+9=\pm\sqrt{180}$$

$$3t+9=\pm\sqrt{36\cdot 5}$$

$$3t+9=\pm\sqrt{36}\sqrt{5}$$

$$3t+9=\pm 6\sqrt{5}$$

$$3t=-9\pm 6\sqrt{5}$$

$$t=\frac{-9\pm 6\sqrt{5}}{3}$$

$$t=-3\pm 2\sqrt{5}$$

The solutions to the equation are:

$-3\pm 2\sqrt{5}$

13. The y-intercept occurs where $x=0$; the y-intercept is $(0,3)$.

The x-intercepts occurs where $y=0$.

$$3x^2-5x+3=0$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(3)}}{2(3)}$$

$$x=\frac{5\pm\sqrt{-11}}{6}$$

The equation $3x^2-5x+3=0$ has no real number solutions, so the parabola $y=3x^2-5x+3$ has no x-intercepts.

14. The vertex occurs where $x=-\frac{b}{2a}=-\frac{5}{2(-1)}=2.5$. The vertex is $(2.5,18.25)$.

The y-intercept occurs where $x=0$; the y-intercept is $(0,12)$.

The x-intercepts occurs where $y=0$.

$$-x^2+5x+12=0$$
$$x^2-5x-12=0$$

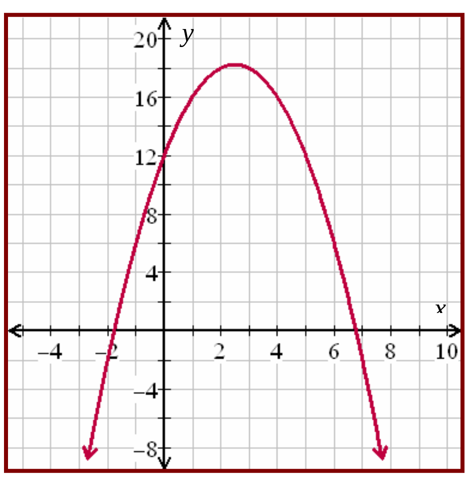
$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-12)}}{2(1)}$$

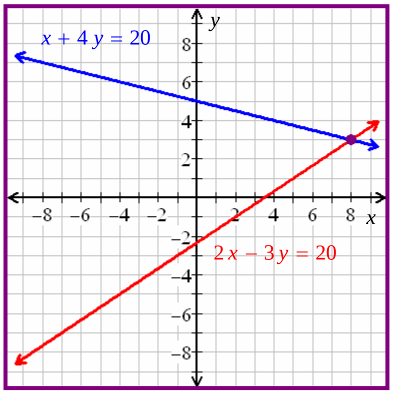
$$x=\frac{5\pm\sqrt{73}}{2}$$

The x-intercepts are:

$\left(\frac{5+\sqrt{73}}{2},0\right)$ and $\left(\frac{5+\sqrt{73}}{2},0\right)$

x	-2	-1	0	1	2	2.5	3	4	5	6	7
y	-2	6	12	16	18	18.25	18	16	12	6	-2



15. 

The solution to the system is $(8,3)$.

16. Let x be the amount ($) that Ms. Hoho had invested in the 6% account and y be the amount ($) she had invested in the 6.5% account. We then have the system:

$$\begin{cases} x + y = 17000 \\ .06x + .065y = 1087 \end{cases}$$

Solving the first equation for y gives us $y = 17000 - x$ and substituting $17000 - x$ for y in the second equation gives us:

$$.06x + .065(17000 - x) = 1087 \Rightarrow .06x + 1105 - .065x = 1087 \Rightarrow -.005x = -18 \Rightarrow x = 3600$$

Substituting 3600 for x gives us $y = 17000 - x = 17000 - 3600 = 13400$.

So Ms. Hoho had $3,600 invested in the 6% account and $13,400 invested in the 6.5% account.

17. Let w be the width (inches) and h the height (inches) of the rectangle the Barnster bent. We then have the system:

$$\begin{cases} \text{Perimeter} = 30 \text{ in} \\ \text{Area} = 55.25 \text{ in}^2 \end{cases} \Rightarrow \begin{cases} 2w + 2h = 30 \\ wh = 55.25 \end{cases}$$

Solving the first equation for h gives us $h = 15 - w$ and substituting $15 - w$ for h in the second equation gives us:

$$w(15 - w) = 55.25 \Rightarrow 15w - w^2 = 55.25 \Rightarrow 0 = w^2 - 15w + 55.25$$

So

$$w = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(55.25)}}{2(1)} \Rightarrow w = \frac{15 \pm \sqrt{4}}{2} \Rightarrow w = \frac{15+2}{2} = 8.5 \text{ or } w = \frac{15-2}{2} = 6.5$$

Lo and behold, $2(8.5) + 2(6.5) = 30$!

So any way you bend it, one side of the Barnster's rectangle is 8.5 inches long and the other is 6.5 inches long.

18. Since we know that the difference in the speeds for Mr. Chugalot's (Charlie's) trip was 10 mph, we should let the variable, r, be the average speed (mph) at which Mr. Chugalot (Charlie) drove to Jumptown; then the speed at which he drove home was $r-10$.

We know that it took $1+\frac{12}{60}$ hours for the Chugster (Charlie) to drive to Jumptown. $1+\frac{12}{60}=1.2$. Using $D=rt$, the distance from the Chuckster's (Charlie's) house to the Buffalo Lodge is $1.2r$.

Similarly, since it took 1.5 hours for Chuck (Charlie) to drive home, the distance from the Buffalo Lodge to Charlie's house is $1.5(r-10)$.

This gives us the mercifully simple equation $1.2r=1.5(r-10) \Rightarrow 1.2r=1.5r-15 \Rightarrow r=50$

It took Charlie 1.2 hours at a speed of 50 mph to drive to the lodge, so the distance from Charlie's house to the lodge is:

$$(1.2 \text{ hours})\left(50\,\frac{\text{miles}}{\text{hour}}\right)=60 \text{ miles}$$

19. a. $g(0)=5$ b. The domain of g is $[-4,\infty)$. c. The range of g is $(-\infty,6]$.