1. a. $$\begin{aligned}(2x+3)(x-5) &= 2x^2-10x+3x-15\\ &= 2x^2-7x-15\end{aligned}$$

b. $$\begin{aligned}(5x-1)^2 &= (5x-1)(5x-1)\\ &= 25x^2-5x-5x+1\\ &= 25x^2-10x+1\end{aligned}$$

c. $$\begin{aligned}(3x+4)(x^2-2x+12) &= 3x^3-6x^2+36x+4x^2-8x+48\\ &= 3x^3-2x^2+28x+48\end{aligned}$$

d. $$\begin{aligned}(x+1)(x-1)(2x-1) &= (x^2-1)(2x-1)\\ &= 2x^3-x^2-2x+1\end{aligned}$$

e. $$\begin{aligned}(x+2)^3 &= (x+2)(x+2)(x+2)\\ &= (x^2+4x+4)(x+2)\\ &= x^3+2x^2+4x^2+8x+4x+8\\ &= x^3+6x^2+12x+8\end{aligned}$$

f. $$-4x^2y^5(-2x-x^4y^6+1) = 8x^3y^5+4x^6y^{11}-4x^2y^5$$

g. $$\begin{aligned}(3x^2y^6+a^2c^4)(3x^2y^6-a^2c^4) &= (3x^2y^6)^2-(a^2c^4)^2\\ &= 9x^4y^{12}-a^4c^8\end{aligned}$$

h. $$\begin{aligned}(x+y)(x^2-xy+y^2)(x^3-y^3) &= (x^3\cancel{-x^2y}\cancel{+xy^2}\cancel{+x^2y}\cancel{-xy^2}+y^3)(x^3-y^3)\\ &= (x^3+y^3)(x^3-y^3)\\ &= (x^3)^2-(y^3)^2\\ &= x^6-y^6\end{aligned}$$

2. a. $$\frac{x^5}{x^{12}} = \frac{1}{x^7}$$

b. $$\begin{aligned}\frac{x^{-12}}{x^{-5}} &= \frac{x^5}{x^{12}}\\ &= \frac{1}{x^7}\end{aligned}$$

c. $$\frac{12a^5b^9}{50a^9b^2} = \frac{6b^7}{25a^4}$$

d. $\dfrac{3^{-1}x^{-2}y^{5}}{3^{2}y}=\dfrac{y^{5}}{3^{1}\cdot 3^{2}x^{2}y}$

$=\dfrac{y^{4}}{27x^{2}}$

e. $\dfrac{5x^{-2}y}{(5x)^{-2}}=\dfrac{5x^{-2}y}{5^{-2}x^{-2}}$

$=\dfrac{5\cdot 5^{2}x^{2}y}{x^{2}}$

$=125y$

f. $\left(\dfrac{-2x^{3}y^{-4}}{x^{8}y^{-2}}\right)^{-1}=\dfrac{(-2)^{-1}x^{-3}y^{4}}{x^{-8}y^{2}}$

$=\dfrac{x^{8}y^{4}}{(-2)^{1}x^{3}y^{2}}$

$=-\dfrac{x^{5}y^{2}}{2}$

g. $\left(-3x^{-3}y^{-1}z\right)^{-2}=(-3)^{-2}x^{6}y^{2}z^{-2}$

$=\dfrac{x^{6}y^{2}}{(-3)^{2}z^{2}}$

$=\dfrac{x^{6}y^{2}}{9z^{2}}$

h. $\left(-\dfrac{7r^{-1}t^{-22}}{rs^{-12}t^{41}}\right)^{0}=1$

i. $\dfrac{\left(4xy^{-1}\right)^{-1}\left(3x^{-1}\right)}{\left(3x^{-1}\right)^{-1}}=\dfrac{4^{-1}x^{-1}y^{1}\cdot 3x^{-1}}{3^{-1}x}$

$=\dfrac{y^{1}\cdot 3\cdot 3^{1}}{4^{1}x^{1}x^{1}x}$

$=\dfrac{9y}{4x^{3}}$

j. $a^{2}b^{5}\cdot\dfrac{a^{7}b^{-2}}{a^{-1}b^{-2}}=\dfrac{a^{2}b^{5}}{1}\cdot\dfrac{a^{7}a^{1}b^{2}}{b^{2}}$

$=a^{10}b^{5}$

3. a. $g(x)=9$

$5-3x=9$

$-3x=4$

$x=-\dfrac{4}{3}$

The solution is $-\dfrac{4}{3}$.

Check:

$g\left(-\dfrac{4}{3}\right)=5-3\left(-\dfrac{4}{3}\right)$

$=5+4$

$=9$ ✓

b. $g(7)=5-3(7)$

$=-16$

4. a. $\dfrac{36x^{7}-14x^{3}+12x^{2}}{12x^{2}}=\dfrac{36x^{7}}{12x^{2}}-\dfrac{14x^{3}}{12x^{2}}+\dfrac{12x^{2}}{12x^{2}}$

$=3x^{5}-\dfrac{7}{6}x+1$

b. $$\frac{-24a^4b^8+16a^3b^3-8a^2b^2}{-8a^2b^2}=\frac{-24a^4b^8}{-8a^2b^2}+\frac{16a^3b^3}{-8a^2b^2}-\frac{8a^2b^2}{-8a^2b^2}$$
$$=3a^2b^6+(-2ab)-(-1)$$
$$=3a^2b^6-2ab+1$$

5. a. $-\frac{4}{x}$ b. 8 c. $4xy$ and $-7x$

d. 7 e. -1 f. $-\frac{1}{5}$

g. 5 h. -1 i. -25

6. $$w(2)=-2^{-2}+\frac{1}{4}$$
$$=-\frac{1}{2^2}+\frac{1}{4}$$
$$=-\frac{1}{4}+\frac{1}{4}$$
$$=0$$

$$w(-2)=-(-2)^{-2}+\frac{1}{4}$$
$$=-\frac{1}{(-2)^2}+\frac{1}{4}$$
$$=-\frac{1}{4}+\frac{1}{4}$$
$$=0$$

7. a. The domain of f is $(-3,\infty)$ and the range of f is $(-\infty,6]$.

b. $f(3)=4$ and $f(-5)$ does not exist. c. $f(x)=-2$ when $x=5$.