Key Concepts: Simplifying square roots
Quadratic Equations/ The square root property
Quadratic Equations/ The quadratic formula

Square roots

- If b is a positive real number, the principal square root of b is the positive number, a, with the property that $a^2 = b$.
- Positive real numbers in fact have two square roots, one positive ($\sqrt{b}$) and one negative ($-\sqrt{b}$).
- $\sqrt{0} = 0$
- The square roots of a negative real number are not real numbers.

Find each square root.

$\sqrt{36}$

$\sqrt{36} = 6$

$-\sqrt{100}$

$-\sqrt{100} = -10$

$\sqrt{\frac{4}{9}}$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$

$-\sqrt{-25}$

$-\sqrt{-25}$ is not a real number

The product rule for square roots

If $a \geq 0$ and $b \geq 0$, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

Simplify each expression.

$\sqrt{50}$

$$\sqrt{50} = \sqrt{25 \cdot 2}$$
$$= \sqrt{25}\sqrt{2}$$
$$= 5\sqrt{2}$$

$-\sqrt{32}$

$$-\sqrt{32} = -\sqrt{16 \cdot 2}$$
$$= -\sqrt{16}\sqrt{2}$$
$$= -4\sqrt{2}$$

$\frac{2 \pm \sqrt{6}}{2}$ Does not simplify

$\frac{2 \pm 1\sqrt{6}}{2}$ The GCF is 2.

$\sqrt{12}\sqrt{3}$

$$\sqrt{12}\sqrt{3} = \sqrt{(12)(3)} = \sqrt{36} = 6$$

$$\sqrt{12}\sqrt{3} = \sqrt{4 \cdot 3}\sqrt{3} = \sqrt{4}\sqrt{3}\sqrt{3} = 2(\sqrt{3})^2 = 2(3) = 6$$

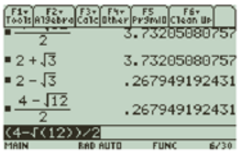
$\sqrt{180}$

$$\sqrt{180} = \sqrt{9 \cdot 20} = \sqrt{9 \cdot 4 \cdot 5} = \sqrt{9}\sqrt{4}\sqrt{5} = (3)(2)\sqrt{5} = 6\sqrt{5}$$

$\frac{4 \pm \sqrt{12}}{2}$

$$\frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm \sqrt{4 \cdot 3}}{2} = \frac{4 \pm \sqrt{4}\sqrt{3}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \frac{4}{2} \pm \frac{2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

This two different numbers $2 + \sqrt{3}$ and $2 - \sqrt{3}$



$\frac{2\sqrt{3}}{2} \Leftarrow$ treat it like you would treat $\frac{2x}{2}$

$\frac{2x}{2} = x$

$\frac{16 \pm \sqrt{27}}{4}$

$$\frac{16 \pm \sqrt{27}}{4} = \frac{16 \pm \sqrt{9 \cdot 3}}{4} = \frac{16 \pm \sqrt{9}\sqrt{3}}{4} = \frac{16 \pm 3\sqrt{3}}{4}$$

Done

Unless those three factors have a GCF > 1, the number does not simplify anymore

$$\frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{3}$$

$$= \sqrt{3}$$

$$\frac{18}{\sqrt{6}} = \frac{18}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{18\sqrt{6}}{6}$$

$$= 3\sqrt{6}$$

$$\frac{1}{\sqrt{8}} = \frac{1}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$$

$$= \frac{\sqrt{8}}{8}$$

$$= \frac{\sqrt{4 \cdot 2}}{8}$$

$$= \frac{\sqrt{4}\sqrt{2}}{8}$$

$$= \frac{2\sqrt{2}}{8}$$

$$= \frac{\sqrt{2}}{4}$$

$$\frac{2x}{8} = \frac{x}{4}$$

Problem set 1
problem set 2
10-1

$(3x+6)^2 = 18$

$(3x+6)^2 = 18$

$3x+6 = \sqrt{18}$ or $3x+6 = -\sqrt{18}$

$3x+6 = \pm\sqrt{18}$

$3x+6 = \pm\sqrt{9\cdot 2}$

$3x+6 = \pm\sqrt{9}\sqrt{2}$

$3x+6 = \pm 3\sqrt{2}$

$3x = -6 \pm 3\sqrt{2}$

$x = \frac{-6}{3} \pm \frac{3\sqrt{2}}{3}$

$x = -2 \pm \sqrt{2}$

The solutions are $-2 \pm \sqrt{2}$.

$-2+\sqrt{2}$ and $-2-\sqrt{2}$

$(5t-7)^2 = 72$

$(5t-7)^2 = 72$

$5t-7 = \sqrt{72}$ or $5t-7 = -\sqrt{72}$

$5t-7 = \pm\sqrt{72}$

$5t-7 = \pm\sqrt{36\cdot 2}$

$5t-7 = \pm\sqrt{36}\sqrt{2}$

$5t-7 = \pm 6\sqrt{2}$

$5t = 7 \pm 6\sqrt{2}$

$t = \frac{7 \pm 6\sqrt{2}}{5}$

The solutions are

$\frac{7+6\sqrt{2}}{5}$ and $\frac{7-6\sqrt{2}}{5}$

$(7w+1)^2 = -81$

$(7w+1)^2 = -81$

The solutions are not real numbers.

$(\text{real number})^2 \geq 0$

extracting the roots

The square root property

If $a \geq 0$ and $u^2 = a$, then $\underbrace{u = \sqrt{a} \text{ or } u = -\sqrt{a}}$

Use the square root property to find all solutions to each equation.

$x^2 = 9$

$x^2 = 9$

$x = \sqrt{9}$ or $x = -\sqrt{9}$

$x = 3$ or $x = -3$

The solutions are 3 and -3.

$t^2 = 44$

$t^2 = 44$

$t = \sqrt{44}$ or $t = -\sqrt{44}$

$t = \sqrt{4 \cdot 11}$ or $t = -\sqrt{4 \cdot 11}$

$t = \sqrt{4}\sqrt{11}$ or $t = -\sqrt{4}\sqrt{11}$

$t = 2\sqrt{11}$ or $t = -2\sqrt{11}$

The solutions are $2\sqrt{11}$ and $-2\sqrt{11}$

$(2x-1)^2 = 9$

$(2x-1)^2 = 9$

$2x - 1 = \sqrt{9}$ or $2x - 1 = -\sqrt{9}$

$2x - 1 = 3$ or $2x - 1 = -3$

$x = 2$ or $x = -1$

The solutions are 2 and -1.

Check

$x = 2$

$(2x-1)^2 = 3^2 = 9$ ✓

$x = -1$

$(2x-1)^2 = (-3)^2 = 9$ ✓

$$5(x-2)^2+3=18$$

$$5(x-2)^2=15$$

$$\frac{5(x-2)^2}{5}=\frac{15}{5}$$

$$(x-2)^2=3$$

$x-2=\sqrt{3}$ or $x-2=-\sqrt{3}$

So $x=2\pm\sqrt{3}$

The solutions are $2+\sqrt{3}$ and $2-\sqrt{3}$.

The quadratic formula

If $a \neq 0$, then the solutions to the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula to solve each equation.

$11x^2 - 10x + 2 = 0$

$a = 11$
$b = -10$
$c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(11)(2)}}{2(11)}$$
$$x = \frac{10 \pm \sqrt{100 - 88}}{22}$$
$$x = \frac{10 \pm \sqrt{12}}{22}$$
$$x = \frac{10 \pm \sqrt{4 \cdot 3}}{22}$$
$$x = \frac{10 \pm \sqrt{4}\sqrt{3}}{22}$$
$$x = \frac{10 \pm 2\sqrt{3}}{22}$$
$$x = \frac{5 \pm \sqrt{3}}{11}$$

Scratch work

$$\frac{\overset{5}{\cancel{10}} \pm \overset{1}{\cancel{2}}\sqrt{3}}{\underset{11}{\cancel{22}}}$$

The solutions are $\frac{5+\sqrt{3}}{11}$ and $\frac{5-\sqrt{3}}{11}$

$2z^2 = z + 3$

$$2z^2 = z + 3$$
$$2z^2 - z - 3 = 0$$

$a = 2$ $\quad c = -3$
$b = -1$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$z = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$
$$z = \frac{1 \pm \sqrt{1 + 24}}{4}$$

$z = \frac{1+\sqrt{25}}{4}$ or $z = \frac{1-\sqrt{25}}{4}$

$z = \frac{1+5}{4}$ or $z = \frac{1-5}{4}$

$z = \frac{3}{2}$ or $z = -1$

The solutions are $\frac{3}{2}$ and -1

$10 - x^2 = 2x$

$$10 - x^2 = 2x$$

$$0 = x^2 + 2x - 10$$

$a = 1$

$b = 2$

$c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{44}}{2}$$

$$x = \frac{-2 \pm \sqrt{4 \cdot 11}}{2}$$

$$x = \frac{-2 \pm \sqrt{4}\sqrt{11}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{11}}{2}$$

$$x = -1 \pm \sqrt{11}$$

The solutions are $-1 \pm \sqrt{11}$

Scratch work

$$\frac{\cancel{-2}^{\,1} \pm \cancel{2}^{\,1}\sqrt{11}}{\cancel{2}\;1}$$

~~$5t^2 - 2t + 1 = 0$~~

$(3t + 1)(2t - 1) = -200$

$$(3t + 1)(2t - 1) = -200$$

$$6t^2 - t - 1 = -200$$

$$6t^2 - t + 199 = 0$$

$a = 6$

$b = -1$

$c = 199$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(199)}}{2(6)}$$

$$t = \frac{1 \pm \sqrt{-4775}}{12}$$

Ao: is outraged by $\sqrt{-4775}$ for good reason!

The solutions are out real numbers

$\frac{y^2}{2} = 3y - 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{y^2}{2} = 3y - 1$$

$$2\left(\frac{y^2}{2}\right) = 2(3y - 1)$$

$$y^2 = 6y - 2$$

$$y^2 - 6y - 2 = 0$$

$$a = 1,\ b = -6,\ c = -2$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$

$$y = \frac{6 \pm \sqrt{44}}{2}$$

$$y = \frac{6 \pm \sqrt{4 \cdot 11}}{2}$$

$$y = \frac{6 \pm \sqrt{4}\sqrt{11}}{2}$$

$$y = \frac{6 \pm 2\sqrt{11}}{2}$$

$$y = 3 \pm \sqrt{11}$$

The solutions are $3 + \sqrt{11}$ and $3 - \sqrt{11}$

$5u - 4u^2 = \frac{3}{2}$

$$5u - 4u^2 = \frac{3}{2}$$

$$2(5u - 4u^2) = 2\left(\frac{3}{2}\right)$$

$$10u - 8u^2 = 3$$

$$0 = 8u^2 - 10u + 3$$

$$a = 8,\ b = -10,\ c = 3$$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$u = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(8)(3)}}{2(8)}$$

$$u = \frac{10 \pm \sqrt{4}}{16}$$

$$u = \frac{10 + 2}{16} \quad \text{OR} \quad u = \frac{10 - 2}{16}$$

$$u = \frac{3}{4} \quad \text{OR} \quad u = \frac{1}{2}$$

The solutions are $\frac{3}{4}$ and $\frac{1}{2}$.

Check this out

$$u = \frac{3}{4} \qquad u = \frac{1}{2}$$

$$4u = 3 \qquad 2u = 1$$

$$\underbrace{4u - 3 = 0 \qquad 2u - 1 = 0}$$

$$(4u - 3)(2u - 1) = 8u^2 - 10u + 3$$

$4x^2 + 4x = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$4x^2 + 4x = -1$

$4x^2 + 4x + 1 = 0$

$a = 4$
$b = 4$
$c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{0}}{8}$$

$x = \frac{-4+0}{8}$ or $x = \frac{-4-0}{8}$

There's only one solution: $-\frac{1}{2}$.

$36x^2 = 15$

(1) It's kind of lame to use the quadratic formula here because

$36x^2 = 15$

$x^2 = \frac{15}{36}$

$x = \sqrt{\frac{15}{36}}$ or $x = -\sqrt{\frac{15}{36}}$

$x = \frac{\sqrt{15}}{6}$ or $x = -\frac{\sqrt{15}}{6}$

(2) If we choose to be lame, we can do the quadratic formula

$36x^2 = 15$

$36x^2 - 15 = 0$

$a = 36$
$b = 0$
$c = -15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(36)(-15)}}{2(36)}$$

$$x = \frac{\pm \sqrt{4}\sqrt{36}\sqrt{15}}{72}$$

$$x = \pm \frac{(2)(6)\sqrt{15}}{72}$$

$$x = \pm \frac{\sqrt{15}}{6}$$

The solutions are $\frac{\sqrt{15}}{6}$ and $-\frac{\sqrt{15}}{6}$.