$x^2 + 7x + 12$

$(x+3)(x+4)$

multiply to 12

12
3 4
7

add to 7

Key Concepts: Quadratic Equations
The zero principle
Yippe-ay-yay... word problems!! ☺

The zero principle

If $ab = 0$, then $a = 0$ and/or $b = 0$.

Examples

Find the solutions to the equation $(3x+8)(5x-7)=0$.

$(3x+8)(5x-7) = 0$

$3x+8=0$ or $5x-7=0$

$x = -\frac{8}{3}$ or $x = \frac{7}{5}$

The solutions are $-\frac{8}{3}$ and $\frac{7}{5}$.

Solve the equation $x(x-6)=0$.

$x(x-6) = 0$

$x=0$ or $x-6=0$

$x=0$ or $x=6$

The solutions are 0 and 6.

Check

If $x=0$

$x(x-6) = (0)(-6) = 0$

If $x=6$

$x(x-6) = (6)(0) = 0$

Solve the equation $x(x-3)=10$.

There is no "ten principle"

There is a zero principle!

$x(x-3) = 10$

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x-5=0$ or $x+2=0$

$x=5$ or $x=-2$

-10
-5 +2
-3

The solutions are 5 and -2.

Check

$x=5$

$x(x-3) = (5)(2) = 10$ ✓

$x=-2$

$x(x-3) = (-2)(-5) = 10$ ✓

The zero principle

- Remember, it's the ***zero*** principle. You cannot use the principle unless one side of the equation is ***zero***.
- Remember, the principle says that if two expressions are ***multiplied*** and the result is zero, the value of at least one of the expressions must be zero. The principle cannot be applied unless the non-zero side of the equation is completely ***factored***.

Find the solutions to the equation $x^3 + 3x^2 - x = 3$

$x^3 + 3x^2 - x = 3$

$x^3 + 3x^2 - x - 3 = 0$

$x^2(x+3) - 1(x+3) = 0$

$(x+3)(x^2-1) = 0$

$(x+3)(x+1)(x-1) = 0$

$x+3=0$ or $x+1=0$ or $x-1=0$

$x=-3$ or $x=-1$ or $x=1$

Check

$x=1$

$1^3+3(1)^2-1=3$?

$1+3-1=3$ ✓

$x=-1$

$(-1)^3+3(-1)^2-(-1)=3$?

$-1+3+1=3$ ✓

$x=-3$

$(-3)^3+3(-3)^2-(-3)=3$?

$-27+27+3=3$ ✓

The solutions are 1, −1, and −3.

Quadratic Equations

Equations that *can* be written in the form $ax^2 + bx + c = 0,\ a \neq 0$ are called ***quadratic equations***. The form $ax^2 + bx + c = 0$ is ***called the standard form of a quadratic equation.***

A strategy for solving quadratic equations

1. Write the equation in standard form. This sometimes requires expanding the product of two binomials not equal to zero.
2. Factor the non-zero side of the equation and apply the zero principle.

7.6

36
+9 +4
13

Solve each equation.

$x^2 + 13x = -36$

$$x^2 + 13x = -36$$
$$x^2 + 13x + 36 = 0$$
$$(x+9)(x+4) = 0$$
$$x + 9 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -9 \quad \text{or} \quad x = -4$$

The solutions are -9 and -4.

Check

$x = -9$

$(-9)^2 + 13(-9) = -36$?

$81 - 117 = -36$ ✓

$x = -4$

$(-4)^2 + 13(-4) = -36$?

$16 - 52 = -36$ ✓

$4t(8t+9) = 5$

$$4t(8t+9) = 5$$
$$32t^2 + 36t = 5$$
$$32t^2 + 36t - 5 = 0$$
$$32t^2 + 40t - 4t - 5 = 0$$
$$8t(4t+5) - 1(4t+5) = 0$$
$$(4t+5)(8t-1) = 0$$

$4t+5=0$ or $8t-1=0$

$t = -\frac{5}{4}$ or $t = \frac{1}{8}$

The solutions are $-\frac{5}{4}$ and $\frac{1}{8}$

$(32)(-5)$

-160

$+40$ -4

36

✓
✓

$y(2y+1) = 0$

$$y(2y+1) = 0$$
$$y = 0 \quad \text{or} \quad 2y + 1 = 0$$
$$y = 0 \quad \text{or} \quad y = -\frac{1}{2}$$

The solutions are 0 and $-\frac{1}{2}$

$(x-10)(x+1) = -10$

$(x-10)(x+1) = -10$

$x^2 - 9x - 10 = -10$

$x^2 - 9x - 10 + 10 = -10 + 10$

$x^2 - 9x = 0$

$x(x-9) = 0$

$x = 0$ or $x - 9 = 0$

$x = 0$ or $x = 9$

The solutions are 0 and 9.

$7(x-8) = x - (x-8)$

$7(x-8) = x - (x-8)$

$7x - 56 = x - x + 8$

$7x - 56 = 8$

$7x = 64$

$x = \frac{64}{7}$

The solution is $\frac{64}{7}$

$0 = 36t^2 - 12t - 35$

$-(2)(3)(2)(3)(7)(5)$

$(36)(-35)$

$-42 \quad 30$

-12

$0 = 36t^2 - 12t - 35$

$0 = 36t^2 - 42t + 30t - 35$

$0 = 6t(6t-7) + 5(6t-7)$

$0 = (6t-7)(6t+5)$

$6t - 7 = 0$ or $6t + 5 = 0$

$6t = 7$ or $6t = 5$

$t = 7/6$ or $t = -5/6$

The solutions are $\frac{7}{6}$ and $-\frac{5}{6}$.

$-8=0$ or $t-7=0$ or $2t+3=0$

never or $t=7$ or $t=-3/2$

A bean bag is thrown into the air from the top of a 168 ft building with an initial velocity of 88 ft/s. It can be shown using calculus that the height of the bag (ft) t seconds after it is shot is given by the function $h(t) = -16t^2 + 88t + 168$. Find the number of seconds it takes for the bag to fall to the ground.

When the bag is on the ground, $h(t)=0$

$$h(t)=0$$

$$-16t^2+88t+168=0$$

$$-8(2t^2-11t-21)=0$$

$$-8\left[2t^2-14t+3t-21\right]=0$$

$$-8\left[2t(t-7)+3(t-7)\right]=0$$

$$-8(t-7)(2t+3)=0$$

-42; -14, $+3$; -11

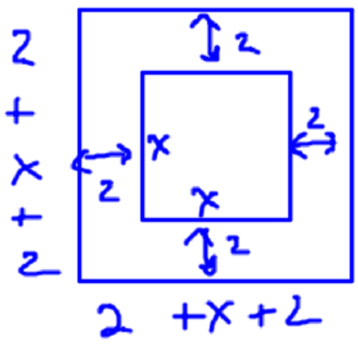
$t-7=0$ or $2t+3=0$

$t=7$ or $t=-3/2$

Umm.. bean bag not on ground before bean bag thrown ($t \neq -3/2$)

∴ The bean bag hit the ground exactly seven seconds after it was thrown.

Swing your partner, 'cuz it's hoe-down time at the Shady Proprietors Retirement Home – the monthly Ronald Reagan memorial square dance, to be precise. There's some serious dosadoing going on, and at the end of the spins each side of the square is 2 ft wider than it was before the call to dosado. The area of the new square is 484 ft^2. What was the area of the square before the dosadoing began?



Let x be the length (ft) of each side of the pre-dosado square, so the length of each side of the post-dosado square is $x+4$

$$(x+4)^2=484$$

$$(x+4)(x+4)=484$$

$$x^2+8x+16=484$$

$$x^2+8x-468=0$$

-468; $+26$, -18; 8

$(x+26)(x-18)=0$

$x+26=0$ or $x-18=0$

$x=-26$ or $x=18$

(nope)

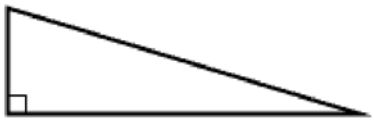
pre-dosado square

18 ft, 18 ft

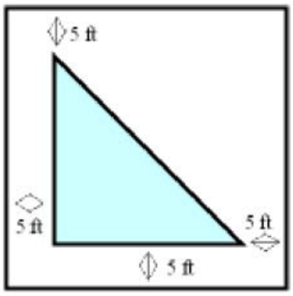
The pre-dosado area was 324 ft^2

468 = 4 · 117; 4 = 2 · 2; 117 = 9 · 13; 9 = 3 · 3

One leg of a right triangle is 7 cm longer than the other and the hypotenuse is 1 cm longer than the longer leg. What are the lengths of each side of the triangle?



Dr. Dieter has a pool in her backyard that is in the shape of an isosceles right triangle! The pool sits in a square tiled walkway as illustrated below. The total area of the pool area (including the walkway) is 2025 ft^2. How long is each side of Dr. Dieter's triangular pool?



Velma Smeltz invested all of her bingo winnings in a government bond. The bond earns an annual interest rate, r, that is applied at the end of each year. The amount of money, A, in Velma's account at the end of the t^{th} year is given by the formula $A = 175(1+r)^t$. At the end of the second year there was \$196.63 in the account. What is the annual interest rate on the account?