Key Concepts: More factoring
Trinomials where the leading coefficient isn't 1
Binomials – some special formulas
Factor problems that just keep on keeping on

Trinomials where the leading coefficient isn't 1 $(ax^2 + bx + c,\ a \neq 1)$

> As we should when factoring any expression, the first thing we should look for is a GFC other than 1. If there is such a beast, yank it out. Make sure that you check to see if the resultant trinomial factor is factorable!

<u>Examples</u>
Factor each expression completely.

$4x^2y - 20xy + 24y$

$-9t^5 + 90xt^4 - 216x^2$

$14x^2y^5 + 56x^2y^4 + 420x^2y^3$

$$ax^2 + bx + c, \ a \neq 1$$

It is frequently the case that the GCF is indeed 1. When this is the case, one tactic you can try is called guess and check. Think of a pair of numbers that multiply to a and a different pair of numbers that multiply to c. Write down possible factorizations and check to see if they FOIL to the original expression. Hmmmmmmmmm….

Use guess and check to factor $4x^2 - 11x + 6$. Include a list of all the possible factorizations.

Use guess and check to factor $5x^2 - 2x - 7$. Include a list of all the possible factorizations.

$$ax^2 + bx + c, \;\; a \neq 1$$ Plan b!!

Find, if it exists, a pair of numbers (h and k) whose product is ac and whose sum is b.

- If such a pair exists, rewrite the polynomial as $ax^2 + hx + kx + c$ and factor by grouping.
- If no such pair exists, the polynomial is prime (assuming, of course, that you didn't forget to start the process by looking for the GCF of all of the terms. ☺)

Factor by grouping: $8x^2 - 14x + 3$.

Factor by grouping: $36x^2 - 48xy + 15y^2$.

Factor by grouping: $9x^6 - 25x^3y^2 - 6y^4$.

Factor $x^{10} - 25y^4$

$$a^2 - b^2 = (a+b)(a-b)$$

$a^2 =$	$b^2 =$
$a =$	$b =$

Factor $8x^3 - y^6$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$a^3 =$	$b^3 =$
$a =$	$b =$

Factor $125t^{12} + 27x^9$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$a^3 =$	$b^3 =$
$a =$	$b =$

Binomials – some special formulas

Binomials – some special formulas

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$

$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

A special fact that you need to incorporate into your reality.

Unless a and b share a common factor other than 1, $a^2 + b^2$ is prime!

Factor, factor, factor!

$36p^2 - q^2$ $\quad$ $36p^2 + q^2$ $\quad$ $125 + y^3$ $\quad$ $8x^3 - 27y^3$

Factor $x^2 + 7x + 12$ and $x^2 + 7xy + 12y^2$

Factor $t^2 - 10t - 24$ and $x^6 - 10x^3 - 24$

Factor $x^2 - 29x + 28$ and $a^{10} - 29a^5b^3 + 28b^6$

Factor problems that just keep on keeping on

Factor, factor, factor … factor, factor … factor …

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Factor $x^8 - y^8$.

Factor $x^6 y^2 - y^8$.

Factor $8x^4 + 10x^2 - 3$.

A Factor Plan

1. **Always** begin by factoring out the GCF of the terms.

2. If there's a binomial lurking about, see if it fits one of the special form.

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

3. If there's a trinomial hanging around:

   - If the leading coefficient is 1 and the trinomial is factorable, then:

     $x^2 + bx + c = (x+h)(x+k)$ where $hk = c$ and $h + k = b$

   - If the leading coefficient isn't 1, you could try "guess and check" or you could try factoring by grouping after finding a pair of numbers (h and k) whose product is ac and whose sum is b and writing:

     $$ax^2 + bx + c = ax^2 + hx + kx + c$$

4. If there's a four-termed polynomial in the room, try to factor by grouping.

5. In all of the above circumstances, **check your answer by multiplying… check your answer by multiplying… check your answer by multiplying… check your answer by multiplying… check your answer by multiplying…**

6. Remember – prime happens. Be open to the possibility but make sure that you've considered and checked all viable factorizations before concluding that the polynomial is prime.