Key Concepts: More factoring
Trinomials where the leading coefficient isn't 1
Binomials – some special formulas
Factor problems that just keep on keeping on

Trinomials where the leading coefficient isn't 1 $(ax^2 + bx + c,\ a \neq 1)$

> As we should when factoring any expression, the first thing we should look for is a GFC other than 1. If there is such a beast, yank it out. Make sure that you check to see if the resultant trinomial factor is factorable!

Examples

Factor each expression completely.

Factor pair: −2, −3

$4x^2y - 20xy + 24y$

$$4x^2y - 20xy + 24y = 4y(x^2 - 5x + 6)$$
$$= 4y(x-2)(x-3)$$

$-9t^5 + 90xt^4 - 216x^2$

$$-9t^5 + 90xt^4 - 216x^2 = -9(t^5 - 10xt^4 + 24x^2)$$

DONE

$14x^2y^5 + 56x^2y^4 + 420x^2y^3$

$$14x^2y^5 + 56x^2y^4 + 420x^2y^3 = 14x^2y^3(y^2 + 4y + 30)$$

Done

Factor pair of 30 that adds to 4?

really?

NO WAY

REALLY?

$$ax^2 + bx + c,\ a \neq 1$$

It is frequently the case that the GCF is indeed 1. When this is the case, one tactic you can try is called guess and check. Think of a pair of numbers that multiply to a and a different pair of numbers that multiply to c. Write down possible factorizations and check to see if they FOIL to the original expression. Hmmmmmmmmm....

Use guess and check to factor $4x^2 - 11x + 6$. Include a list of all the possible factorizations.

x $4x$ or $2x$ $2x$ -1 -6 or -2 -3

look Possible F & L for factors & trying the possibilities until O + I = −11

	OI	
$(x-1)(4x-6)$	$-6x - 4x$	Bad
$(x-6)(4x-1)$	$-x - 24x$	Bad
$(x-2)(4x-3)$	$-3x - 8x$	✓✓✓✓
$(x-3)(4x-2)$		
$(2x-1)(2x-6)$		
$(2x-2)(2x-3)$		

$4x^2 - 11x + 6 = (x-2)(4x-3)$

Use guess and check to factor $5x^2 - 2x - 7$. Include a list of all the possible factorizations.

$5x$ x -7 1 / 7 -1

	OI
$(5x-7)(x+1)$	$5x - 7x$ ✓✓
$(5x+1)(x-7)$	
$(5x+7)(x-1)$	
$(5x-1)(x+7)$	

$5x^2 - 2x - 7 = (5x-7)(x+1)$

$x^2+2x-3=0$
$(x+3)(x-1)=0$
The solutions are -3 and 1

$$ax^2+bx+c,\ a\neq 1 \qquad \text{Plan b!!}$$

Find, if it exists, a pair of numbers (h and k) whose product is ac and whose sum is b.

- If such a pair exists, rewrite the polynomial as $ax^2+hx+kx+c$ and factor by grouping.
- If no such pair exists, the polynomial is prime (assuming, of course, that you didn't forget to start the process by looking for the GCF of all of the terms. ☺)

Factor by grouping: $8x^2-14x+3$.

We need a factor pair that multiplies to $(8)(3)=24$ and adds to -14
How's about -2 + -12

$$8x^2-14x+3=8x^2-12x-2x+3$$
$$=4x(2x-3)-1(2x-3)$$
$$=(2x-3)(4x-1)$$

$12(5)=60$

Factor by grouping: $36x^2-48xy+15y^2$.

$$36x^2-48xy+15y^2=3(12x^2-16xy+5y^2)$$

We need a factor pair that multiplies to 60 and adds to -16 $(-)(-)$
-6 + -10

$$=3[12x^2-6xy-10xy+5y^2]$$
$$=3[6x(2x-y)-5y(2x-y)]$$
$$=3(2x-y)(6x-5y)$$

Factor by grouping: $9x^6-25x^3y^2-6y^4$.

Holy Cannoli, I need a factor pair of -54 that adds to -25
$-27, 2$

$$9x^6-25x^3y^2-6y^4$$
$$=9x^6-27x^3y^2+2x^3y^2-6y^4$$
$$=9x^3(x^3-3y^2)+2y^2(x^3-3y^2)$$
$$=(x^3-3y^2)(9x^3+2y^2)$$

$x^8 - 36$ $4r^2 - 81t^6$

$a^2 = x^8$ $b^2 = 36$
$a =$ $b =$

Factor $x^{10} - 25y^4$

$$a^2 - b^2 = (a+b)(a-b)$$

$a^2 = x^{10}$ $b^2 = 25y^4$

$a = x^5$ $b = 5y^2$

$x^{10} - 25y^4 = (x^5 + 5y^2)(x^5 - 5y^2)$

Factor $8x^3 - y^6$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$a^3 = 8x^3$ $b^3 = y^6$

$a = 2x$ $b = y^2$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^2 = 4x^2$ $b^2 = y^4$

$ab = 2xy^2$

$8x^3 - y^6 = (2x - y^2)(4x^2 + 2xy^2 + y^4)$

Check

$(2x - y^2)(4x^2 + 2xy^2 + y^4) = 8x^3 + 4x^2y^2 + 2xy^4 - 4x^2y^2 - 2xy^4 - y^6 = 8x^3 - y^6$

Factor $125t^{12} + 27x^9$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$a^3 = 125t^{12}$ $b^3 = 27x^9$

$a = 5t^4$ $b = 3x^3$

$a^2 = 25t^8$ $b^2 = 9x^6$

$ab = (5t^4)(3x^3)$
$= 15x^3t^4$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$125t^{12} + 27x^9 = (5t^4 + 3x^3)(25t^8 - 15x^3t^4 + 9x^6)$

Check

$125t^{12} - \cancel{75x^3t^8} + \cancel{45x^6t^4} + \cancel{75x^3t^8} - \cancel{45x^6t^4} + 27x^9$

Binomials – some special formulas

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$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

A special fact that you need to incorporate into your reality.

Unless a and b share a common factor other than 1, $a^2 + b^2$ is prime!

Factor, factor, factor!

$36p^2 - q^2$ $\quad$ $36p^2 + q^2$ $\quad$ $125 + y^3$ $\quad$ $8x^3 - 27y^3$

$36p^2 - q^2 = (6p+q)(6p-q)$

$36p^2 + q^2$ <u>is</u> Prime! Accept it!!

$125 + y^3 = (5+y)(25 - 5y + y^2)$

$8x^3 - 27y^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$

Factor $x^2 + 7x + 12$ and $x^2 + 7xy + 12y^2$

$$x^2 + 7x + 12 = (x+4)(x+3)$$

$$\begin{aligned} x^2 + 7xy + 12y^2 &= x^2 + 3xy + 4xy + 12y^2 \\ &= x(x+3y) + 4y(x+3y) \\ &= (x+3y)(x+4y) \end{aligned}$$

Factor $t^2 - 10t - 24$ and $x^6 - 10x^3 - 24$

$$t^2 - 10t - 24 = (t-12)(t+2)$$

$$\begin{aligned} x^6 - 10x^3 - 24 &= x^6 - 12x^3 + 2x^3 - 24 \\ &= x^3(x^3-12) + 2(x^3-12) \\ &= (x^3-12)(x^3+2) \end{aligned}$$

Factor $x^2 - 29x + 28$ and $a^{10} - 29a^5b^3 + 28b^6$

$$x^2 - 29x + 28 = (x-1)(x-28)$$

$$\begin{aligned} a^{10} - 29a^5b^3 + 28b^6 &= a^{10} - 28a^5b^3 - a^5b^3 + 28b^6 \\ &= a^5(a^5 - 28b^3) - b^3(a^5 - 28b^3) \\ &= (a^5 - b^3)(a^5 - 28b^3) \end{aligned}$$

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incorporate into their reality base

$(a+b)^2 \neq a^2+b^2$

Factor problems that just keep on keeping on

Factor, factor, factor … factor, factor … factor …

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$
$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

Factor $x^8 - y^8$.

Factor $x^6 y^2 - y^8$.

Factor $8x^4 + 10x^2 - 3$.

$$x^8 - y^8 = (x^4+y^4)(x^4-y^4)$$
$$= (x^4+y^4)(x^2+y^2)(x^2-y^2)$$
$$= (x^4+y^4)(x^2+y^2)(x+y)(x-y)$$

$a^2 = x^8 \quad b^2 = y^8$

$a = x^4 \quad b = y^4$

$(a+b)(a-b)$

$a^2 = x^4 \quad b^2 = y^4$

$a = x^2 \quad b = y^2$

$$x^6y^2 - y^8 = y^2(x^6 - y^6)$$
$$= y^2(x^3+y^3)(x^3-y^3)$$
$$= y^2(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$

new problem

$a^2 = x^6 \quad b^2 = y^6$

$a = x^3 \quad b = y^3$

sum/diff of cubes

$a = x \quad b = y$

$$8x^4 + 10x^2 - 3 = 8x^4 + 12x^2 - 2x^2 - 3$$
$$= 4x^2(2x^2+3) - 1(2x^2+3)$$
$$= (2x^2+3)(4x^2-1)$$
$$= (2x^2+3)(2x+1)(2x-1)$$

-24

$12 \quad -2$

next problem

$a^2 = 4x^2 \quad b^2 = 1$

$a = 2x \quad b = 1$

A Factor Plan

1. Always begin by factoring out the GCF of the terms.

2. If there's a binomial lurking about, see if it fits one of the special form.

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

3. If there's a trinomial hanging around:

   - If the leading coefficient is 1 and the trinomial is factorable, then:

     $x^2 + bx + c = (x+h)(x+k)$ where $hk = c$ and $h + k = b$

   - If the leading coefficient isn't 1, you could try "guess and check" or you could try factoring by grouping after finding a pair of numbers (h and k) whose product is ac and whose sum is b and writing:

$$ax^2 + bx + c = ax^2 + hx + kx + c$$

4. If there's a four-termed polynomial in the room, try to factor by grouping.

5. In all of the above circumstances, check your answer by multiplying… check your answer by multiplying… check your answer by multiplying… check your answer by multiplying… check your answer by multiplying…

6. Remember - prime happens. Be open to the possibility but make sure that you've considered and checked all viable factorizations before concluding that the polynomial is prime.