Key Concepts: Factoring
Greatest Common Factor
Factor by grouping
Factor trinomials of form $x^2 + bx + c$

Finding the greatest common factor of polynomial terms

The greatest common factor of two or more polynomial terms is the product of:

- The greatest common factor of the coefficients of the terms (which is frequently 1)

and

- Each and every variable that appears in *every one of the terms* raised to the *smallest power* found on the variable in *any one term*.

Examples

Find the greatest common factor of each group of terms.

$6x^5y^3z,\ -8x^4yz^2,\ -10x^7z^7$

① GCF of 6, 8 & 10 is 2

② x is in every term
smallest exponent is 4

③ y is not in every term

④ z is in every term
smallest exponent is 1

The GCF is $\underline{\underline{2x^4z}}$.

$75a^8b^2,\ 90a^8b^5$

The GCF is $15a^8b^2$

$10x^2y,\ 15x^7z^2, -8y^8z^9$

The GCF 1.

Examples

Factor from each polynomial the greatest common factor of all of the terms

$28xy^2 - 21xy$

$$28xy^2 - 21xy = 7xy(4y - 3)$$

$9a^6b^9 - 12a^5b^4 + 3a^3b^3$

$$9a^6b^9 - 12a^5b^4 + 3a^3b^3 = 3a^3b^3(3a^3b^6 - 4a^2b + 1)$$

$-70u^5v^8 - 35u^2v^{10}$

$$-70u^5v^8 - 35u^2v^{10} = -35u^2v^8(2u^3 + v^2)$$

$-32x^2 - 40x + 24$

$$-32x^2 - 40x + 24 = -8(4x^2 + 5x - 3)$$

$x(x+7) - 8(x+7)$

$$x\overbrace{(x+7)}^{B} - 8\overbrace{(x+7)}^{B} = \overbrace{(x+7)}^{B}(x - 8)$$

$y(y-5) - (y-5)$

$$y(y-5) - (y-5) = (y-5)(y-1)$$

Now you try

Factor the greatest common factor from each polynomial.

a. $x^2 y^3 + 2xy^2 - 3x^4 y^2$

b. $18a^7 b^9 - 60a^7 b^6 - 6a^7 b^4$

c. $3x^2 - 6x^8 + 6x^{25}$

d. $-72u^{10} v^{12} - 48uw^9 - 90v^7 w$

e. $x(x+8) + (x+8)$

f. $3x^2(4x^2 - 9) - 4(4x^2 - 9)$

g. $14x^2 y - 21xz^3 + 6yz$

a. $x^2y^3 + 2xy^2 - 3x^4y^2 = xy^2(xy + 2 - 3x^3)$

b. $18a^7b^9 - 60a^7b^6 - 6a^7b^4 = 6a^7b^4(3b^5 - 10b^2 - 1)$

c. $3x^2 - 6x^8 + 6x^{25} = 3x^2(1 - 2x^6 + 2x^{23})$

d. $-72u^{10}v^{12} - 48uw^9 - 90v^7w$
$= -6(12u^{10}v^{12} + 8uw^9 + 15v^7w)$

e. $x(x+8) + (x+8) = (x+8)(x+1)$

f. $3x^2(4x^2 - 9) - 4(4x^2 - 9) = (4x^2 - 9)(3x^2 - 4)$

g. $14x^2y - 21xz^3 + 6yz$ is prime.

Factoring by grouping

Factoring by grouping is a technique applied to polynomials with 4 terms. The trick is to factor something from the first two terms and something different from the final two terms so that the binomials left behind are the same in each factorization. You can then factor the binomial from the resultant two terms.

Examples

Factor each of the following by grouping

$x^2 + 5x - 3x - 15$

$$x^2 + 5x - 3x - 15 = x(x+5) - 3(x+5)$$
$$= (x+5)(x-3)$$

$8x + 50 + 4xy + 25y$

$$8x + 50 + 4xy + 25y = 2(4x+25) + y(4x+25)$$
$$= (4x+25)(2+y)$$

$4a^2 + 10a - 10a - 25$

$$4a^2 + 10a - 10a - 25 = 2a(2a+5) - 5(2a+5)$$
$$= (2a+5)(2a-5)$$

$w^2 - 6wz - 7wz + 42z^2$

$$w^2 - 6wz - 7wz + 42z^2 = w(w-6z) - 7z(w-6z)$$
$$= (w-6z)(w-7z)$$

Now you try

Factor each polynomial by grouping.

a. $xy + 5y - 2x - 10$

b. $x^2 - 11x - 9x + 99$

c. $3x^2 - 12x + 21x - 28$

d. $4y^2 - 36xy - 7xy + 63x^2$

a. $xy + 5y - 2x - 10 = y(x+5) - 2(x+5)$

$= (x+5)(y-2)$

b. $x^2 - 11x - 9x + 99 = x(x-11) - 9(x-11)$

$= (x-11)(x-9)$

c. $3x^2 - 12x + 21x - 28$ is prime

d. $4y^2 - 36xy - 7xy + 63x^2 = 4y(y-9x) - 7x(y-9x)$

$= (y-9x)(4y-7x)$

Factoring trinomials of form $x^2 + bx + c$

- Find, if they exist, two integers, h and k, that multiply to c and add to b.
- If the numbers exist, then $x^2 + bx + c = (x+h)(x+k)$.
- If no two such numbers exist then the trinomial is prime.

Examples

Factor each trinomial. factor pair of 1 & 6

$x^2 + 7x + 6$

$$x^2+7x+6 = x^2 + x + 6x + 6$$
$$= x(x+1) + 6(x+1)$$
$$= (x+1)(x+6)$$

Factor pairs of 6

Factor pair	Sum
1, 6	7
2, 3	5
−1, −6	−7
−2, −3	−5

$x^2 - 5x + 6$

$$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$$
$$= x(x-2) - 3(x-2)$$
$$= (x-2)(x-3)$$

$x^2 - x - 6$

$$x^2 - x - 6 = x^2 + 2x - 3x - 6$$
$$= x(x+2) - 3(x+2)$$
$$= (x+2)(x-3)$$

Factor pairs of −6

Factor pair	Sum
1, −6	−5
−1, 6	5
−2, 3	1
2, −3	−1

$x^2 + 7x - 6$

$x^2 + 7x - 6$ is prime.

1. 7.1
2. Pinksheet problem sets 1-4
3. 7.2
4. Graded 4a

$y^2 - 9y - 90$

$$y^2 - 9y - 90 = y^2 - 15y + 6y - 90$$
$$= y(y-15) + 6(y-15)$$
$$= (y-15)(y+6)$$

relevant factor pairs

+ / − ← "bigger #"

Pair	Sum
−90, 1	−89
−45, 2	−43
−30, 3	−27
−18, 5	−13
−15, 6	−9 ✓

$t^2 - 16t + 60$

$$t^2 - 16t + 60 = t^2 - 6t - 10t + 60$$
$$= t(t-6) - 10(t-6)$$
$$= (t-6)(t-10)$$

relevant factor pairs

− / −

Pair	Sum
−60, −1	−61
−30, −2	−32
−20, −3	−23
−15, −4	−19
−12, −5	−17
−10, −6	Ahh!!!

$w^2 + 8w + 80$

we need +/+ factors

$w^2 + 8w + 80$ is prime

(At least one of the numbers in any factor pair of 80 is already bigger than 8

Now you try

Factor each trinomial by trial and error.

a. x^2+6x+5 **b.** t^2-6t+8 **c.** x^2+8x+4

d. $y^2-14y-32$ **e.** $x^2+6x-20$ **f.** $x^2+15x-100$

g. $w^2-18w+45$ **h.** $x^2+21x+90$ **i.** $b^2-9b+400$

a. $x^2+6x+5 = x^2+5x+x+5$
$= x(x+5)+1\cdot(x+5)$
$= (x+5)(x+1)$

b. $t^2-6t+8 = t^2-4t-2t+8$
$= t(t-4)-2(t-4)$
$= (t-4)(t-2)$

c. x^2+8x+4 is prime!

d. $y^2-14y-32 = y^2-16y+2y-32$
$= y(y-16)+2(y-16)$
$= (y-16)(y+2)$

e. $x^2+6x-20$ is prime

f. $x^2+15x-100 = x^2+20x-5x-100$
$= x(x+20)-5(x+20)$
$= (x+20)(x-5)$

g. $w^2-18w+45 = w^2-15w-3w+45$
$= w(w-15)-3(w-15)$
$= (w-15)(w-3)$

h. $x^2+21x+90 = x^2+15x+6x+90$
$= x(x+15)+6(x+15)$
$= (x+15)(x+6)$

i. $b^2-9b+400$ is so prime!

Key Concepts: Why do we factor?
Solving equations – the zero principle
Simplifying rational expressions

The zero principle

If $ab = 0$, then $a = 0$ and/or $b = 0$.

Examples

Find the solutions to the equation $(x+8)(x-10)=0$.

Solve the equation $x^2 + 4x - 45 = 0$.

Simplifying rational expressions

Reduce the fraction $\dfrac{21+12}{7+2}$.

Correct	Wrong

Simplify the rational expression $\dfrac{x^2-5x-14}{x^2-15x+56}$.

Correct	Wrong