Problem Set 1

a. $x^2+12x+32 = x^2+4x+8x+32$
$= x(x+4)+8(x+4)$
$= (x+8)(x+4)$

b. $y^2+38y+72 = y^2+2y+36y+72$
$= y(y+2)+36(y+2)$
$= (y+36)(y+2)$

c. $x^2+18x+72 = x^2+6x+12x+72$
$= x(x+6)+12(x+6)$
$= (x+12)(x+6)$

d. $x^2+30x+72$ is prime

e. $a^2+14a+49 = (a+7)(a+7)$
$= (a+7)^2$

f. $y^2+91y+90 = (y+90)(y+1)$

g. $t^2+29t+180 = (t+9)(t+20)$

h. $a^2+4a+24$ is prime

Problem Set 2

a. $x^2-14x+48 = x^2-6x-8x+48$
$= x(x-6)-8(x-6)$
$= (x-8)(x-6)$

b. $b^2-27b+92 = b^2-4b-23b+92$
$= b(b-4)-23(b-4)$
$= (b-23)(b-4)$

c. $x^2-29x+92$ is prime

d. $y^2-26y+48 = (y-24)(y-2)$

e. $x^2-16x+60 = (x-10)(x-6)$

f. $a^2-30a+81 = (a-27)(a-3)$

g. $t^2-27t+27$ is prime

h. $m^2-30m+225 = (m-15)(m-15)$
$= (m-15)^2$

Problem Set 3

a. $t^2+9t-90 = t^2-6t+15t-90$
$= t(t-6)+15(t-6)$
$= (t+15)(t-6)$

b. $t^2+5t-24 = t^2-3t+8t-24$
$= t(t-3)+8(t-3)$
$= (t+8)(t-3)$

c. $z^2+15z-100 = z^2-5z+20z-100$
$= z(z-5)+20(z-5)$
$= (z+20)(z-5)$

d. $y^2+y-90 = (y+10)(y-9)$

e. $m^2 + 20m - 44 = (m + 22)(m - 2)$

f. $p^2 + 200p - 100$ is prime

g. $x^2 + 4x - 24$ is prime

h. $w^2 + 22w - 240 = (w - 8)(w + 30)$

Problem Set 4

a. $x^2 - 6x - 16 = x^2 + 2x - 8x - 16$
$= x(x + 2) - 8(x + 2)$
$= (x - 8)(x + 2)$

b. $t^2 - 16t - 80 = t^2 + 4t - 20t - 80$
$= t(t + 4) - 20(t + 4)$
$= (t - 20)(t + 4)$

c. $p^2 - 11p - 80 = p^2 + 5p - 16p - 80$
$= p(p + 5) - 16(p + 5)$
$= (p - 16)(p + 5)$

d. $y^2 - 2y - 48 = (y - 8)(y + 6)$

e. $x^2 - 20x - 48$ is prime

f. $w^2 - 3w - 180 = (w - 15)(w + 12)$

g. $z^2 - 88z - 180 = (z - 90)(z + 2)$

h. $w^2 - 399w - 400 = (w - 400)(w + 1)$

Problem Set 5

a. $10x^2 + 39x + 14 = 10x^2 + 4x + 35x + 14$
$= 2x(5x + 2) + 7(5x + 2)$
$= (2x + 7)(5x + 2)$

b. $14w^2 - 31w - 10 = 14w^2 + 4w - 35w - 10$
$= 2w(7w + 2) - 5(7w + 2)$
$= (2w - 5)(7w + 2)$

c. $12y^2 - 19y + 4 = 12y^2 - 3y - 16y + 4$
$= 3y(4y - 1) - 4(4y - 1)$
$= (3y - 4)(4y - 1)$

d. $6y^2 + 47y - 8 = 6y^2 - y + 48y - 48$
$= y(6y - 1) + 48(y - 1)$
$= (y + 48)(6y - 1)$

e. $6b^2 + 13b - 8 = 6b^2 - 3b + 16b - 8$
$= 3b(2b - 1) + 8(2b - 1)$
$= (3b + 8)(2b - 1)$

f. $4t^2 - 17t + 4 = 4t^2 - 16t - t + 4$
$= 4t(t - 4) - 1(t - 4)$
$= (4t - 1)(t - 4)$

g. $4x^2 + 12x + 9 = 4x^2 + 6x + 6x + 9$
$= 2x(2x + 3) + 3(2x + 3)$
$= (2x + 3)(2x + 3)$
$= (2x + 3)^2$

h. $8w^2 + 7w - 1 = 8w^2 + 8w - w - 1$
$= 8w(w + 1) - (w + 1)$
$= (8w - 1)(w + 1)$

i. $8x^2 - 14x - 15 = 8x^2 - 20x + 6x - 15$
$= 4x(2x - 5) + 3(2x - 5)$
$= (4x + 3)(2x - 5)$

j. $27m^2 + 21m + 2 = 27m^2 + 3m + 18m + 2$
$= 3m(9m + 1) + 2(9m + 1)$
$= (3m + 2)(9m + 1)$

Problem Set 6

a. $x^4 - 2x^2y - 99y^2 = x^4 + 9x^2y - 11x^2y - 99y^2$
$= x^2(x^2 + 9y) - 11y(x^2 + 9y)$
$= (x^2 - 11y)(x^2 + 9y)$

b. $14w^2 - 31w - 10 = 14w^2 + 4w - 35w - 10$
$= 2w(7w + 2) - 5(7w + 2)$
$= (2w - 5)(7w + 2)$

c. $32 + 4w^{20} - w^{40} = 32 - 4w^{20} + 8w^{20} - w^{40}$
$= 4(8 - w^{20}) + w^{20}(8 - w^{20})$
$= (4 + w^{20})(8 - w^{20})$

d. $15x^2 - 16xy + 4y^2 = 15x^2 - 6xy - 10xy + 4y^2$
$= 3x(5x - 2y) - 2y(5x - 2y)$
$= (3x - 2y)(5x - 2y)$

e. $x^2 + 15xy^2 + 36y^4 = x^2 + 3xy^2 + 12xy^2 + 36y^4$
$= x(x + 3y^2) + 12y^2(x + 3y^2)$
$= (x + 12y^2)(x + 3y^2)$

f. $4a^2 - 24ab + 35b^2 = 4a^2 - 10ab - 14ab + 35b^2$
$= 2a(2a - 5b) - 7b(2a - 5b)$
$= (2a - 7b)(2a - 5b)$

g. $1-59w^8-60w^{16}=1+w^8-60w^8-60w^{16}$
$=1(1+w^8)-60w^8(1+w^8)$
$=(1-60w^8)(1+w^8)$

h. $2t^4-7t^2-30=2t^4+5t^2-12t^2-30$
$=t^2(2t^2+5)-6(2t^2+5)$
$=(t^2-6)(t^2+5)$

i. $t^{10}+16t^5+64=t^{10}+8t^5+8t^5+64$
$=t^5(t^5+8)+8(t^5+8)$
$=(t^5+8)(t^5+8)$
$=(t^5+8)^2$

j. $15m^2n^2+8mn+1=15m^2n^2+3mn+5mn+1$
$=3mn(5mn+1)+1(5mn+1)$
$=(3mn+1)(5mn+1)$

Problem Set 7

a. $x^2-81=(x-9)(x+9)$

b. $4w^2-9y^2=(2w-3y)(2w+3y)$

c. $100-49x^{10}=(10-7x^5)(10+7x^5)$

d. $y^{42}-25x^2=(y^{21}-5x)(y^{21}+5x)$

e. $49x^2-1=(7x-1)(7x+1)$

f. $121-9w^4=(11-3w^2)(11+3w^2)$

g. $36x^{18}-25y^6=(6x^9-5y^3)(6x^9+5y^3)$

Problem Set 8

a. $t^3-y^6=(t-y^2)\left[t^2+ty^2+(y^2)^2\right]$
$=(t-y)(t^2+ty^2+y^4)$

b. $27-x^3=(3-x)\left[3^2+3x+x^2\right]$
$=(3-x)(9+3x+x^2)$

c. $8x^3+27y^3=(2x+3y)\left[(2x)^2-(2x)(3y)+(3y)^2\right]$
$=(2x+3y)(4x^2-6xy+9y^2)$

d. $216+125m^6=\left(6+5m^2\right)\left[6^2-6\left(5m^2\right)+\left(5m^2\right)^2\right]$
$=\left(6+5m^2\right)\left(36-30m^2+25m^4\right)$

e. $8t^3-1=(2t-1)\left[(2t)^2+(2t)(1)+1^2\right]$
$=(2t-1)\left(4t^2+2t+1\right)$

f. $x^9+1000=\left(x^3+10\right)\left[\left(x^3\right)^2+\left(x^3\right)(10)+10^2\right]$
$=\left(x^3+10\right)\left(x^6+10x^3+100\right)$

g. $27x^6+125y^3=\left(3x^2+5y\right)\left[\left(3x^2\right)^2-\left(3x^2\right)(5y)+(5y)^2\right]$
$=\left(3x^2+5y\right)\left(9x^4-15x^2y+25y^2\right)$

Problem Set 9

a. x^2+81 is prime b. $4w^2+9y^2$ is prime c. $100+49x^{10}$ is prime

d. y^4+25x^2 is prime e. $49x^2+49=49\left(x^2+1\right)$ f. $121+9w^4$ is prime

Problem Set 10

a. $3x^2-33x+84=3\left(x^2-11x+28\right)$
$=3(x-4)(x-7)$

b. $32w^4-18w^2=2w^2\left(16w^2-9\right)$
$=2w^2(4w-3)(4w+3)$

c. $120x^4+120x^3-90x^2=30x^2\left(4x^2+4x-3\right)$
$=30x^2\left[4x^2-2x+6x-3\right]$
$=30x^2\left[2x(2x-1)+3(2x-1)\right]$
$=30x^2(2x+3)(2x-1)$

d. $60t^7w-320t^5w+100t^3w=20t^3w\left(3t^4-16t^2+5\right)$
$=20t^3w\left[3t^4-15t^2-t^2+5\right]$
$=20t^3w\left[3t^2\left(t^2-5\right)-1\left(t^2-5\right)\right]$
$=20t^3w\left[\left(3t^2-1\right)\left(t^2-5\right)\right]$
$=20t^3w\left(3t^2-1\right)\left(t^2-5\right)$

e. $a^3 b^6 + 8b^3 = b^3\left(a^3 + 8\right)$

$= b^3\left[(a+2)\left(a^2 + (a)(2) + 2^2\right)\right]$

$= b^3(a+2)\left(a^2 + 2a + 4\right)$

f. $15x^2 + 45x + 15 = 15\left(x^2 + 3x + 1\right)$

g. $x^6 y^4 z - 32x^4 y^5 z + 60x^2 y^6 z = x^2 y^4 z\left(x^4 - 32x^2 y + 60y^2\right)$

$= x^2 y^4 z\left(x^4 - 2x^2 y - 30x^2 y + 60y^2\right)$

$= x^2 y^4 z\left[x^2\left(x^2 - 2y\right) - 30y\left(x^2 - 2\right)\right]$

$= x^2 y^4 z\left(x^2 - 30y\right)\left(x^2 - 2\right)$

h. $x^4 y^2 + 9x^2 y^2 = x^2 y^2\left(x^2 + 9\right)$