Problem Set 1

Each of the following expressions have form $x^2 + bx + c$ where b and c are both positive numbers. Our goal is to find a pair of integers that multiply to c and add to b. Since, in these problems, both b and c are both positive, each number in the successful factor pair will have to be positive.

Example: Factor $x^2 + 19x + 48$

Possible factor pairs: $1, 48$ or $2, 24$ or $3, 16$ or $4, 12$ or $6, 8$

The pair that adds to 19 is $3, 16$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $19x$ term is split into $3x + 16x$.

$$\begin{aligned} x^2 + 19x + 48 &= x^2 + 3x + 16x + 48 \\ &= x(x+3) + 16(x+3) \\ &= (x+16)(x+3) \end{aligned}$$

Please note that our successful factor pair was $+16$ and $+3$ and the factorization ended up being $(x+16)(x+3)$.

Example: Factor $m^2 + 80m + 156$

Possible factor pairs: $1, 156$ or $2, 78$ or $3, 52$ or $4, 39$ or $6, 26$ or $12, 13$

The pair that adds to 80 is $2, 78$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $80m$ term is split into $2m + 78m$.

$$\begin{aligned} m^2 + 80m + 156 &= m^2 + 2m + 78m + 156 \\ &= m(m+2) + 78(m+2) \\ &= (m+78)(m+2) \end{aligned}$$

Please note that our successful factor pair was $+78$ and $+2$ and the factorization ended up being $(m+78)(m+2)$.

Example: Factor $y^2 + 21y + 100$

Possible factor pairs: $1, 100$ or $2, 50$ or $4, 25$ or $5, 20$ or $10, 10$

None of the factor pairs adds to 21, so the trinomial factors no further. What you write is

$y^2 + 21y + 100$ is prime.

Now you work the following problems. Use factoring by grouping when working the first 2 or 3 problems and then try leaving those two steps out on the remainder of the problems in this set.

a. Factor $x^2 + 12x + 32$
(Possible factor pairs: $1, 32$ or $2, 16$ or $4, 8$)

b. Factor $y^2 + 38y + 72$
(Possible factor pairs: $1, 72$ or $2, 36$ or $3, 24$ or $4, 18$ or $6, 12$ or $8, 9$)

c. Factor $x^2 + 18x + 72$

d. Factor $x^2 + 30x + 72$

e. Factor $a^2 + 14a + 49$

f. Factor $y^2 + 91y + 90$

g. Factor $t^2 + 29t + 180$

h. Factor $a^2 + 4a + 24$

Problem Set 2

Each of the following expressions have form $x^2 + bx + c$ where c is a positive number and b is a negative number. Our goal is to find a pair of integers that multiply to c and add to b. Since, in these problems, c is positive and b is a negative, each number in the successful factor pair will have to be negative.

Example: Factor $x^2 - 14x + 48$

Possible factor pairs: $-1, -48$ or $-2, -24$ or $-3, -16$ or $-4, -12$ or $-6, -8$

The pair that adds to -14 is $-6, -8$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $-14x$ term is split into $-6x - 8x$.

$$\begin{aligned} x^2 - 14x + 48 &= x^2 - 6x - 8x + 48 \\ &= x(x-6) - 8(x-6) \\ &= (x-8)(x-6) \end{aligned}$$

Please note that our successful factor pair was -8 and -6 and the factorization ended up being $(x-8)(x-6)$.

Example: Factor $y^2 - 26y + 156$

Possible factor pairs: $-1, -156$ or $-2, -78$ or $-3, -52$ or $-4, -39$ or $-6, -26$ or $-12, -13$

None of the factor pairs adds to -26, so the trinomial factors no further. What you write is

$y^2 - 26y + 156$ is prime.

Example: Factor $w^2 - 20w + 100$

Possible factor pairs: $-1,-100$ or $-2,-50$ or $-4,-25$ or $-5,-20$ or $-10,-10$

The pair that adds to -20 is $-10,-10$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $-20w$ term is split into $-10w - 10w$.

$$\begin{aligned} w^2 - 20w + 100 &= w^2 - 10w - 10w + 100 \\ &= w(w-10) - 10(w-10) \\ &= (w-10)(w-10) \\ &= (w-10)^2 \end{aligned}$$

Please note that our successful factor pair was -10 and -10 and the factorization ended up being $(w-10)(w-10)$.

Now you work the following problems. Use factoring by grouping when working the first 2 or 3 problems and then try leaving those two steps out on the remainder of the problems in this set.

a. Factor $x^2 - 14x + 48$
(Possible factor pairs: $-1,-48$ or $-2,-24$ or $-3,-16$ or $-4,-12$ or $-6,-8$)

b. Factor $b^2 - 27b + 92$
(Possible factor pairs: $-1,-92$ or $-2,-46$ or $-4,-23$)

c. Factor $x^2 - 29x + 92$

d. Factor $y^2 - 26y + 48$

e. Factor $x^2 - 16x + 60$

f. Factor $a^2 - 30a + 81$

g. Factor $t^2 - 27t + 27$

h. Factor $m^2 - 30m + 225$

Problem Set 3

Each of the following expressions have form $x^2 + bx + c$ where c is a negative number and b is a positive number. Our goal is to find a pair of integers that multiply to c and add to b. Since, in these problems, c is negative, the numbers in the factor pair will have opposite signs. Since b is positive, the positive number in the successful factor pair will have the greater absolute value.

Example: Factor $y^2 + 47y - 48$

Possible factor pairs: $-1,48$ or $-2,24$ or $-3,16$ or $-4,12$ or $-6,8$

The pair that adds to 47 is $-1,48$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $47y$ term is split into $-y + 48y$.

$$\begin{aligned} y^2 + 47y - 48 &= y^2 - y + 48y - 48 \\ &= y(y-1) + 48(y-1) \\ &= (y+48)(y-1) \end{aligned}$$

Please note that our successful factor pair was $+48$ and -1 and the factorization ended up being $(y+48)(y-1)$.

Now you work the following problems. Use factoring by grouping when working the first 2 or 3 problems and then try leaving those two steps out on the remainder of the problems in this set.

a. Factor $t^2 + 9t - 90$
(Possible factor pairs: $-1, 90$ or $-2, 45$ or $-3, 30$ or $-6, 15$ or $-9, 10$)

b. Factor $t^2 + 5t - 24$
(Possible factor pairs: $-1, 24$ or $-2, 12$ or $-3, 8$ or $-4, 6$)

c. Factor $z^2 + 15z - 100$ d. Factor $y^2 + y - 90$ e. Factor $m^2 + 20m - 44$

f. Factor $p^2 + 200p - 100$ g. Factor $x^2 + 4x - 24$ h. Factor $w^2 + 22w - 240$

Problem Set 4

Each of the following expressions have form $x^2 + bx + c$ where c is a negative number and b is a negative number. Our goal is to find a pair of integers that multiply to c and add to b. Since, in these problems, c is negative, the numbers in the factor pair will have opposite signs. Since b is negative, the negative number in the successful factor pair will have the greater absolute value.

Example: Factor $a^2 - 21a - 100$

Possible factor pairs: $1, -100$ or $2, -50$ or $4, -25$ or $5, -20$ or $10, -10$ (hmm … not really this last one)

The pair that adds to -21 is $4, -25$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $-21a$ term is split into $+4a - 25a$.

$$\begin{aligned} a^2 - 21w - 100 &= a^2 + 4a - 25a - 100 \\ &= a(a+4) - 25(a+4) \\ &= (a-25)(a+4) \end{aligned}$$

Please note that our successful factor pair was -25 and $+4$ and the factorization ended up being $(a-25)(a+4)$.

Now you work the following problems. Use factoring by grouping when working the first 2 or 3 problems and then try leaving those two steps out on the remainder of the problems in this set.

a. Factor $x^2 - 6x - 16$
(Possible factor pairs: $1,-16$ or $2,-8$ or $4,-4$)

b. Factor $t^2 - 16t - 80$
(Possible factor pairs: $1,-80$ or $2,-40$ or $4,-20$ or $5,-16$ or $8,-10$)

c. Factor $p^2 - 11p - 80$

d. Factor $y^2 - 2y - 48$

e. Factor $x^2 - 20x - 48$

f. Factor $w^2 - 3w - 180$

g. Factor $z^2 - 88z - 180$

h. Factor $w^2 - 399w - 400$

Problem Set 5

In the following problems the form of the trinomial is $ax^2 + bx + c$ where $a \neq 1$. In this case, we a factor pair that multiplies to ac and adds to b.

Example: Factor $6x^2 - 5x - 25$

We need a factor pair that multiplies to $(6)(-25) = -150$. Since their product is negative, the numbers in the factor pair will have to have opposite signs. Since they need to add to -5, the negative number in the successful factor pair will have the greater absolute value.

Possible factor pairs: $1,-150$ or $2,-75$ or $3,-50$ or $5,-30$ or $6,-25$ or $10,-15$

The pair that adds to -5 is $10,-15$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $-5x$ term is split into $+10x - 15x$.

$$\begin{aligned} 6x^2 - 5x - 25 &= 6x^2 + 10x - 15x - 25 \\ &= 2x(3x+5) - 5(3x+5) \\ &= (2x-5)(3x+5) \end{aligned}$$

Example: Factor $36p^2 - 12p + 1$

We need a factor pair that multiplies to $(36)(1) = 36$. Since their product is positive, the numbers in the factor pair will have to have the same sign. Since they need to add to -12, each number in the pair will have to be negative

Possible factor pairs: $-1,-36$ or $-2,-18$ or $-3,-12$ or $-4,-9$ or $-6,-6$

The pair that adds to -12 is $-6, -6$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $-12p$ term is split into $-6p - 6p$.

$$\begin{aligned} 36p^2 - 12p + 1 &= 36p^2 - 6p - 6p + 1 \\ &= 6p(6p-1) - 1(6p-1) \\ &= (6p-1)(6p-1) \\ &= (6p-1)^2 \end{aligned}$$

Example: Factor $3x^2 + 7x - 5$

We need a factor pair that multiplies to $(3)(-5) = -15$. Since their product is negative, the numbers in the factor pair will have to have opposite signs. Since they need to add to $+7$, the positive number in the successful factor pair will have the greater absolute value.

Possible factor pairs: $-1, 15$ or $-3, 5$

Since neither of the possible factor pairs adds to $+7$, $3x^2 + 7x - 5$ is prime.

Now you work the following problems. When the leading coefficient (a) isn't 1, it's best to always go ahead and use factoring by grouping.

a. Factor $10x^2 + 39x + 14$
(Relevant factor pairs of 140: $1, 140$ or $2, 70$ or $4, 35$ or $5, 28$ or $7, 20$ or $10, 14$)

b. Factor $14w^2 - 31w - 10$
(Relevant factor pairs of -140: $1, -140$ or $2, -70$ or $4, -35$ or $5, -28$ or $7, -20$ or $10, -14$)

c. Factor $12y^2 - 19y + 4$
(Relevant factor pairs of 48: $-1, -48$ or $-2, -24$ or $-3, -16$ or $-4, -12$ or $-6, -8$)

d. Factor $6y^2 + 47y - 8$
(Relevant factor pairs of -48: $-1, 48$ or $-2, 24$ or $-3, 16$ or $-4, 12$ or $-6, 8$)

e. Factor $6b^2 + 13b - 8$

f. Factor $4t^2 - 17t + 4$

g. Factor $4x^2 + 12x + 9$

h. Factor $8w^2 + 7w - 1$

i. Factor $8x^2 - 14x - 15$

j. Factor $27m^2 + 21m + 2$

Problem Set 6

In this problem set, we have trinomials where someone has played tricks on the variables. The key to successfully factoring these type trinomials is to remember that the factor pairs you're looking for are just like if you had something of form $a x^2 + b x + c$ and that if you use factoring by grouping the funky variables will take care of themselves.

Example: Factor $x^2 y^4 + 26 x y^2 + 48$

We need a factor pair that multiplies to 48 and adds to 26; both numbers in the pair need to be positive.

Possible factor pairs: $1, 48$ or $2, 24$ or $3, 16$ or $4, 12$ or $6, 8$

The pair that adds to 26 is $2, 24$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $+26 x y^2$ term is split into $+2 x y^2 + 24 x y^2$.

$$\begin{aligned} x^2 y^4 + 26 x y^2 + 48 &= x^2 y^4 + 2 x y^2 + 24 x y^2 + 48 \\ &= x y^2 \left(x y^2 + 2\right) + 24\left(x y^2 + 2\right) \\ &= \left(x y^2 + 24\right)\left(x y^2 + 2\right) \end{aligned}$$

Example: Factor $3x^2 - 10 x y^2 - 8 y^4$

We need a factor pair that multiplies to -24 and adds to -10; the numbers in the pair need to have opposite signs and the negative number needs to have the greater absolute value.

Possible factor pairs: $1, -24$ or $2, -12$ or $3, -8$ or $4, -6$

The pair that adds to -10 is $2, -12$. To use the factor by grouping technique, we want to take the original trinomial and rewrite it so that the $-10 x y^2$ term is split into $+2 x y^2 - 12 x y^2$.

$$\begin{aligned} 3x^2 - 10 x y^2 - 8 y^4 &= 3x^2 + 2 x y^2 - 12 x y^2 - 8 y^4 \\ &= x\left(3x + 2 y^2\right) - 4 y^2\left(3x + 2 y^2\right) \\ &= \left(x - 4 y^2\right)\left(3x + 2 y^2\right) \end{aligned}$$

Now you work the following problems. When the variables are funked up, it's best to always go ahead and use factoring by grouping.

a. Factor $x^4 - 2x^2 y - 99 y^2$
(Relevant factor pairs of -99: $1, -99$ or $3, -33$ or $9, -11$)

b. Factor $4a^{12} + 8a^6 b^3 + 3b^6$
(Relevant factor pairs of 12: $1, 12$ or $2, 6$ or $3, 4$)

c. Factor $32 + 4w^{20} - w^{40}$
(Relevant factor pairs of -32: $-1, 32$ or $-2, 16$ or $-4, 8$)

d. Factor $15x^2 - 16xy + 4y^2$
(Relevant factor pairs of 60: $-1, -60$ or $-2, -30$ or $-3, -20$ or $-4, -15$ or $-5, -12$ or $-6, -10$)

e. Factor $x^2 + 15xy^2 + 36y^4$ f. Factor $4a^2 - 24ab + 35b^2$ g. Factor $1 - 59w^8 - 60w^{16}$

h. Factor $2t^4 - 7t^2 - 30$ i. Factor $t^{10} + 16t^5 + 64$ j. Factor $15m^2n^2 + 8mn + 1$

Problem Set 7

Each of the following problems deal with the difference of squares. A difference of squares **always** factors according to the pattern $a^2 - b^2 = (a-b)(a+b)$.

Example: Factor $9x^2 - 100y^2$

This is a difference of squares with $a = 3x$ and $b = 10y$.

$$9x^2 - 100y^2 = (3x - 10y)(3x + 10y)$$

Example: Factor $625 - 16m^{20}$

This is a difference of squares with $a = 25$ and $b = 4m^{10}$.

$$625 - 16m^{20} = (25 - 4m^{10})(25 + 4m^{10})$$

Now you work the following problems.

a. Factor $x^2 - 81$ using $a = x$ and $b = 9$ b. Factor $4w^2 - 9y^2$ using $a = 2w$ and $b = 3y$

c. Factor $100 - 49x^{10}$ using $a = 10$ and $b = 7x^5$

d. Factor $y^{42} - 25x^2$ using $a = y^{21}$ and $b = 5x$

e. Factor $49x^2 - 1$ f. Factor $121 - 9w^4$ g. Factor $36x^{18} - 25y^6$

Problem Set 8

Each of the following problems deal with the sum or difference of cubes. A sum of cubes **always** factors according to the pattern $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and a difference of cubes **always** factors according to the pattern $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

Example: Factor $8m^3 - p^9$

This is a difference of cubes with $a = 2m$ and $b = p^3$.

$$\begin{aligned} 8m^3 - p^9 &= (2m - p^3)\left[(2m)^2 + (2m)(p^3) + (p^3)^2\right] \\ &= (2m - p^3)(4m^2 + 2mp^3 + p^6) \end{aligned}$$

Example: Factor $125w^6 + 27z^3$

This is a sum of cubes with $a = 5w^2$ and $b = 3z$.

$$\begin{aligned} 125w^6 + 27z^3 &= (5w^2 + 3z)\left[(5w^2)^2 - (5w^2)(3z) + (3z)^2\right] \\ &= (5w^2 + 3z)(25w^4 - 15w^2 z + 9z^2) \end{aligned}$$

Now you work the following problems.

a. Factor $t^3 - y^6$ using $a = t$ and $b = y^2$

b. Factor $27 - x^3$ using $a = 3$ and $b = x$

c. Factor $8x^3 + 27y^3$ using $a = 2x$ and $b = 3y$

d. Factor $216 + 125m^6$ using $a = 6$ and $b = 5m^2$

e. Factor $8t^3 - 1$

f. Factor $x^9 + 1000$

g. Factor $27x^6 + 125y^3$

Problem Set 9

The following problems involve sums of squares. **Unless** the GCF of the pair is something other than one or the pair is ***also*** a sum of cubes, **sums of squares never factor**.

Example: Factor $x^2 + 4$

Solution: $x^2 + 4$ is prime.

Think about $x^2 + 0x + 4$. What two positive numbers multiply to 4 and add to 0?

Example: Factor $81x^2 + 16m^4$

Solution: $81x^2 + 16m^4$ is prime.

Example: Factor $100y^4 + 25y^2$

Solution: Even though $100y^4 + 25y^2$ is a sum of squares, it does factor because the GCF between the two terms is not 1.

$$100y^4 + 25y^2 = 25y^2\left(4y^2 + 1\right)$$

Now you work the following problems.

a. Factor $x^2 + 81$ b. Factor $4w^2 + 9y^2$ c. Factor $100 + 49x^{10}$

d. Factor $y^4 + 25x^2$ e. Factor $49x^2 + 49$ f. Factor $121 + 9w^4$

Problem Set 10

Ironically, now that you've practiced all of the other factoring rules, we need to return to the first rule of factoring – if the GCF is not 1, that is the first thing you need to factor out. Make sure that you remember to see if the factor you leave behind factors further.

Example: Factor $-8x^4 + 20x^3y + 48x^2y^2$

$$\begin{aligned}-8x^4 + 20x^3y + 48x^2y^2 &= -4x^2\left(2x^2 - 5xy - 12y^2\right)\\ &= -4x^2\left[2x^2 + 3xy - 8xy - 12y^2\right]\\ &= -4x^2\left[x(2x+3y) - 4y(2x+3y)\right]\\ &= -4x^2(x-4y)(2x+3y)\end{aligned}$$

Now you work the following problems.

a. Factor $3x^2 - 33x + 84$ b. Factor $32w^4 - 18w^2$

c. Factor $120x^4 + 120x^3 - 90x^2$ d. Factor $60t^7w - 320t^5w + 100t^3w$

e. Factor $a^3b^6 + 8b^3$ f. Factor $15x^2 + 45x + 15$

g. Factor $x^6y^4z - 32x^4y^5z + 60x^2y^6z$ h. Factor $x^4y^2 + 9x^2y^2$