

Key Concepts: Negative exponents Division by Monomials

Example 1

- a. Simplify $\frac{x^2}{x^7}$ by expanding the numerator and denominator and dividing out factors of 1.
- b. Use the rule $\frac{a^m}{a^n} = a^{m-n}$ to simplify $\frac{x^2}{x^7}$.
- c. What's the only logical conclusion?

a. $\frac{x^2}{x^7} = \frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$
 $= \frac{x}{x} \cdot \frac{x}{x} \cdot \frac{1}{x \cdot x \cdot x \cdot x \cdot x}$
 $= \frac{1}{x^5}$

b. $\frac{x^2}{x^7} = x^{2-7}$
 $= x^{-5}$

c. Holy smokes!
 $\frac{x^2}{x^7} = \frac{1}{x^5}$

Example 2

Simplify by expanding the numerator and denominator and dividing out factors of 1.

- a. Simplify $\frac{y}{y^4}$ by expanding the numerator and denominator and dividing out factors of 1.
- b. Use the rule $\frac{a^m}{a^n} = a^{m-n}$ to simplify $\frac{y}{y^4}$.
- c. What's the only logical conclusion?

a. $\frac{y}{y^4} = \frac{y}{y y y y}$
 $= \frac{y}{y} \cdot \frac{1}{y y y}$
 $= \frac{1}{y^3}$

b. $\frac{y}{y^4} = \frac{y^1}{y^4}$
 $= y^{1-4}$
 $= y^{-3}$

c. $y^{-3} = \frac{1}{y^3}$

Complete the pattern in Table 1.

Table 1: Powers of 2

2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

$$2^0 = 1 \quad 2^{-1} = \frac{1}{2} \quad 2^{-2} = \frac{1}{2^2} \quad 2^{-3} = \frac{1}{2^3}$$

$$\text{If } x=2 \quad ; \quad 2x^6 = 2 \cdot (2^6) \\ = 2 \cdot 64$$

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Completely simplify each expression. Make sure that your final expression has no negative exponents.

a. t^{-6}

$$t^{-6} = \frac{1}{t^6}$$

b. $2t^{-6}$

$$2t^{-6} = 2 \cdot t^{-6} \\ = 2 \cdot \frac{1}{t^6} \\ = \frac{2}{t^6}$$

c. $-t^{-6}$

$$-t^{-6} = -\frac{1}{t^6}$$

c. $x^{-9}x^5$ (Work this one with rules of exponents and by canceling factors.)

$$\begin{aligned} x^{-9}x^5 &= x^{-9+5} = x^{-4} \\ \hline x^{-9}x^5 &= \frac{1}{x^9} \cdot x^5 \\ &= \frac{\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}} \\ &= \frac{1}{x^4} \end{aligned}$$

$$\begin{aligned} (2t)^{-6} &= \frac{1}{(2t)^6} \\ &= \frac{1}{2^6 t^6} \\ &= \frac{1}{64t^6} \end{aligned}$$

d. -4^2

$$\begin{aligned} -4^2 &= -1 \cdot 4^2 \\ &= -1 \cdot 16 \\ &= -16 \end{aligned}$$

e. 4^{-2}

$$\begin{aligned} 4^{-2} &= \frac{1}{4^2} \\ &= \frac{1}{16} \end{aligned}$$

f. -4^{-2}

$$\begin{aligned} -4^{-2} &= -\frac{1}{4^2} \\ &= -\frac{1}{16} \end{aligned}$$

g. $(-4)^{-2}$

$$\begin{aligned} (-4)^{-2} &= \frac{1}{(-4)^2} \\ &= \frac{1}{(-4)(-4)} \\ &= \frac{1}{16} \end{aligned}$$

Find each value.

3^{-2}

$$3^{-2} = \frac{1}{3^2}$$

$$= \frac{1}{9}$$

-7^{-1}

$$-7^{-1} = -\frac{1}{7^1}$$

$$= -\frac{1}{7}$$

10^{-5}

$$10^{-5} = \frac{1}{10^5}$$

$$= \frac{1}{100000}$$

$$-\left(\frac{3}{2}\right)^4 = -\frac{81}{16}$$

$\frac{1}{2^{-8}}$

$$\frac{1}{2^{-8}} = \frac{2^8}{1}$$

$$= \frac{256}{1}$$

$$= 256$$

$\left(-\frac{2}{3}\right)^{-4}$

$$\left(-\frac{2}{3}\right)^{-4} = \left(-\frac{3}{2}\right)^4$$

$$= \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)$$

$$= \frac{81}{16}$$

$-4^0 - 4^{-2}$

$$\left. \begin{array}{l} \left(-\frac{2}{3}\right)^{-4} = \left(-\frac{3}{2}\right)^4 \\ = \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right) \\ = \frac{81}{16} \end{array} \right\} \begin{array}{l} -4^0 - 4^{-2} \\ = -1 - \frac{1}{4^2} \\ = -1 - \frac{1}{16} \\ = -\frac{17}{16} \end{array}$$

Completely simplify each expression. Make sure that your final expression contains no negative exponents.

$3x^2y^{-1}$

$$3x^2y^{-1} = \frac{3x^2}{y}$$

$3y^{-1}$

$$3y^{-1} = \frac{3}{y}$$

$$xy^{-1} = \frac{x}{y^1}$$

$$= \frac{x}{y}$$

$$\frac{4x^2y^7}{12x^8y} = \frac{y^6}{3x^6}$$

$$\frac{4x^2y^7}{12x^{10}y}$$

$$\left. \begin{aligned} \frac{4x^2y^7}{12x^{10}y} &= \frac{4}{12} \cdot \frac{x^2}{x^{10}} \cdot \frac{y^7}{y} \\ &= \frac{1}{3} \cdot \frac{1}{x^{10-2}} \cdot \frac{y^{7-1}}{1} \\ &= \frac{y^6}{3x^8} \end{aligned} \right\}$$

$$(3y)^{-1}$$

$$\frac{4x^2y^7}{12x^{10}y} = \frac{4}{12} \cdot \frac{x^2}{x^{10}} \cdot \frac{y^7}{y}$$

$$= \frac{1}{3} x^{2-10} y^{7-1}$$

$$= \frac{1}{3} x^{-8} y^6$$

$$= \frac{1}{3} \cdot \frac{1}{x^8} \cdot y^6$$

$$= \frac{y^6}{3x^8}$$

$$\left. \begin{aligned} (2y)^{-1} &= \frac{1}{(2y)^1} \\ &= \frac{1}{2y} \end{aligned} \right\}$$

$$\frac{3^{-2}x^{-4}y^{12}}{x^{-8}y^{16}}$$

$$\frac{-4a^{-8}}{b^{-9}}$$

$$\frac{-4a^{-8}}{b^{-9}} = \frac{-4b^9}{a^8}$$

$$\begin{aligned} \frac{3^{-2}x^{-4}y^{12}}{x^{-8}y^{16}} &= \frac{x^8y^{12}}{3^2x^4y^{16}} \\ &= \frac{x^4}{9y^4} \end{aligned}$$

$$(x+y)^{-2}$$

$$\begin{aligned} (x+y)^{-2} &= \frac{1}{(x+y)^2} \\ &= \frac{1}{(x+y)(x+y)} \end{aligned}$$

$$= \frac{1}{x^2+2xy+y^2}$$

$$\begin{aligned} \frac{3^{-2}x^{-4}y^{12}}{x^{-8}y^{16}} &= 3^{-2}x^{-4-(-8)}y^{12-16} \\ &= 3^{-2}x^4y^{-4} \\ &= \frac{1}{3^2} \cdot \frac{x^4}{1} \cdot \frac{1}{y^4} \\ &= \frac{x^4}{9y^4} \end{aligned}$$

$$\begin{aligned}
 (3a^{-2}b^4)^{-2} &= 3^{-2}a^4b^{-8} \\
 &= \frac{a^4}{3^2b^8} \\
 &= \frac{a^4}{9b^8}
 \end{aligned}$$

$$\frac{(4x^{-1}y^5)^{-3}(x^2y^{-8})^0}{(2xy^2)^{-6}} = x^{(-1)(-3)},$$

$$\begin{aligned}
 \frac{(4x^{-1}y^5)^{-3}(x^2y^{-8})^0}{(2xy^2)^{-6}} &= \frac{4^{-3}x^3y^{-15}(1)}{2^{-6}x^{-6}y^{-12}} \\
 &= \frac{2^6x^3x^6y^{12}}{4^3y^{15}} \\
 &= \frac{64x^9}{64y^3} \\
 &= \frac{x^9}{y^3}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{5x^{-8}z}{4^{-1}x^{14}z^{-3}} \right)^{-2} &= \frac{5^{-2}x^{16}z^{-2}}{4^2x^{-28}z^6} \\
 &= \frac{x^{16}x^{28}}{4^2 \cdot 5^2 z^2 z^6} \\
 &= \frac{x^{44}}{16 \cdot 25 z^8} \\
 &= \frac{x^{44}}{400z^8}
 \end{aligned}$$

Perform the division $\frac{22x^7 - 11x^4 + 55x^2}{11x^2}$.

$$\frac{22x^7 - 11x^4 + 55x^2}{11x^2} = \frac{22x^7}{11x^2} - \frac{11x^4}{11x^2} + \frac{55x^2}{11x^2}$$

$$= 2x^5 - x^2 + 5$$

Divide $25y^{10} + 35y^7 - 50y^6$ by $5y^5$.

$$\frac{25y^{10} + 35y^7 - 50y^6}{5y^5} = \frac{25y^{10}}{5y^5} + \frac{35y^7}{5y^5} - \frac{50y^6}{5y^5}$$

$$= 5y^5 + 7y^2 - 10y$$

Perform the division $\frac{84x^9 + 34x^8 + 40x^7 + 4x^6}{4x^6}$

$$\frac{84x^9 + 34x^8 + 40x^7 + 4x^6}{4x^6} = \frac{84x^9}{4x^6} + \frac{34x^8}{4x^6} + \frac{40x^7}{4x^6} + \frac{4x^6}{4x^6}$$

$$= 21x^3 + \frac{17}{2}x^2 + 10x + 1$$

$$\frac{\cancel{34}^{17}}{4} \frac{17}{2}$$