Key Concepts: Functions
Determining Function Values
Equations that use Function Notation
Domain and Range

Functions

Recall that a set of ordered pairs, $\{(x,y)\}$ defines *y as a function of x* if and only if no two ordered pairs in the set share the same x-coordinate.

If the name of the function is f and the ordered pair (a,b) is in the set, then we say that $f(a)=b$. In this case we say that *the function value at a is b*.

Sometimes the value of the function can be determined from a formula. When a formula is given for a function f, the function value at any given number is determined by *simultaneously* replacing x with that value on both sides of the equal sign; you then simplify the resultant expression on the right side of the equal sign.

Example 1

Find each of the following values of the function f shown in Figure 1. Use appropriate function notation to state each answer.

a. Find $f(1)$.

$f(1) = -5$

b. What is the function value at -2?

$f(-2) = -4$

c. What is the value of f at 0?

$f(0) = 5$

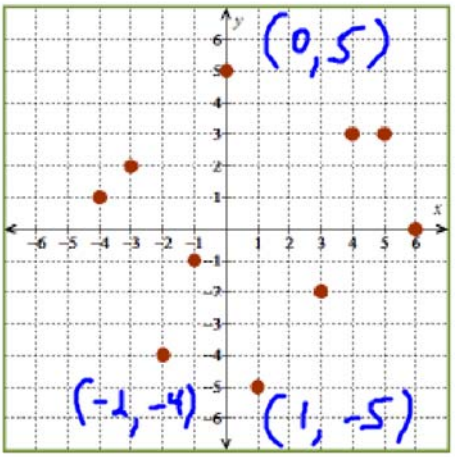

Figure 1: f

d. What is the value of $f(2)$?

option A) $f(2)$ does not exist

option B) $f(2)$ is undefined

Solve $x+4=7$

The solution is 3.

because $3+4=7$.

$3+4=7$

Example 2

Answer each of the following questions about the function g shown in Figure 2.

a. Find $g(-3)$.

$g(-3)=4$

b. What is the function value at -6?

$g(-6)=6$

c. Estimate the value of g at 2.

$g(2)\approx .7$

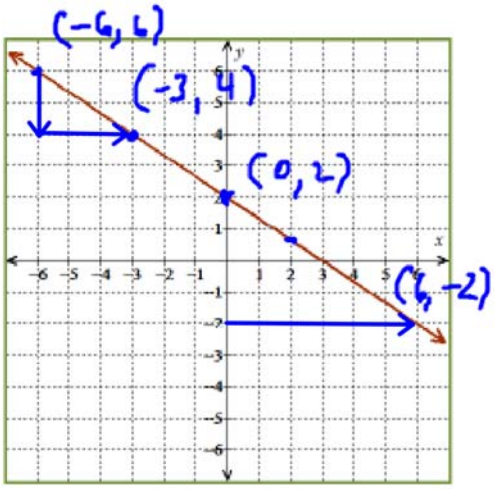

Figure 2: g

d. What is the solution to the equation $g(x)=-2$?

(I need to find the point(s) where $y=-2$)

The solution is 6. (Because $g(6)=-2$ is true)

e. Find the equation of the line in Figure 2; write the equation in slope-intercept form.

$m=-\frac{2}{3}$

y-int: $(0,2)$

The equation of the line is $y=-\frac{2}{3}x+2$

f. What is the formula for $g(x)$? Use that formula to confirm your answers to questions a-d.

$y=g(x)$, so if $y=-\frac{2}{3}x+2$ it's just gotta' be the case that $g(x)=-\frac{2}{3}x+2$.

Check this out!

$g(-3)=-\frac{2}{3}(-3)+2$

$=4$ ✓

$g(-6)=-\frac{2}{3}(-6)+2$

$=\frac{12}{3}+2$

$=6$ ✓

Solve $g(x)=-2$

$-\frac{2}{3}x+2=-2$

$-\frac{3}{2}\left(-\frac{2}{3}x\right)=(-4)\left(-\frac{3}{2}\right)$

$x=6$

The solution is 6.

Example 3

Answer each of the following questions about the function w shown in Figure 3.

a. Find $w(2)$.

$w(2) = 3$

b. What is the function value at -1?

$w(-1) = -3$

c. What is the value of w at -5.

$w(-5) = -11$

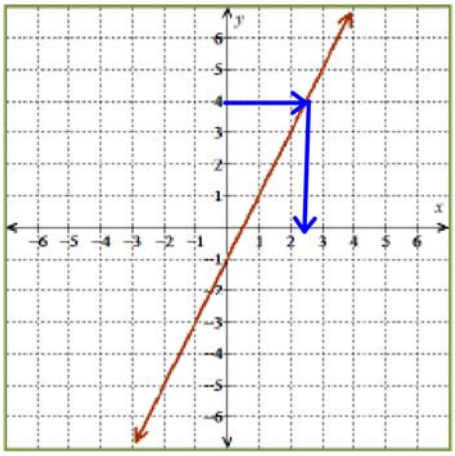

Figure 3: w

d. Estimate the solution to the equation $w(x) = 4$?

The solution is about 2.4.

e. Find the equation of the line in Figure 3; write the equation in slope-intercept form.

$y = 2x - 1$

f. What is the formula for $w(x)$? Use that formula to confirm your answers to questions a-d.

$w(x) = 2x - 1$

$w(2) = 2(2) - 1 = 3$

$w(-1) = 2(-1) - 1 = -3$

$w(-5) = 2(-5) - 1 = -11$

$w(x) = 4$

$2x - 1 = 4$

$2x = 5$

$x = 2\frac{1}{2}$ ✓

Example 4

Percy was taking an oral exam. He was shown the graph in Figure 4 and asked to state the function value at 3. Percy's response was "yo, dawg ... this problem is all whacked." Just what is it about the problem that is all whacked?

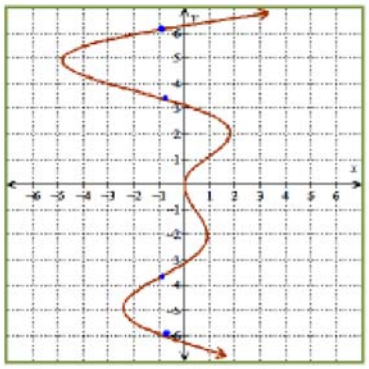

Figure 4: Percy's Problem

Yo, dawg ... this curve ain't no function. There are, for example, four friggin points where $x=-2$.

Example 6

Answer each of the following questions.

a. Find the value of z at -2 if $z(x)=x^2+7$.

$$z(-2) = (-2)^2 + 7$$
$$= 4 + 7$$
$$= 11$$

b. Find the value of $f(6)$ if $f(x)=|x-5|-|1-x|-(x+2)$

$$f(6) = |6-5| - |1-6| - (6+2)$$
$$= |1| - |-5| - 8$$
$$= 1 - 5 - 8$$
$$= -12$$

c. Solve the equation $h(x)=0$ if $h(x)=(x-4)(x-7)$.

$$x^2 - 11x + 28 = 0$$
$$(x-4)(x-7) = 0$$

$$h(x) = 0$$
$$(x-4)(x-7) = 0$$
$$x-4 = 0 \text{ or } x-7 = 0$$
$$x = 4 \text{ or } x = 7$$

The solutions are 4 and 7.

Domain and Range

The domain of the function, $\{(x, y)\}$ is the set of all x-coordinates in the function.

The range of the function, $\{(x, y)\}$ is the set of all y-coordinates in the function.

Example 7

What are the domain and range of the function shown in Figure 5?

Domain: $\{-4, -3, -2, -1, 0, 1, 3, 4, 5, 6\}$

Range: $\{-5, -4, -2, -1, 0, 1, 2, 3, 5\}$

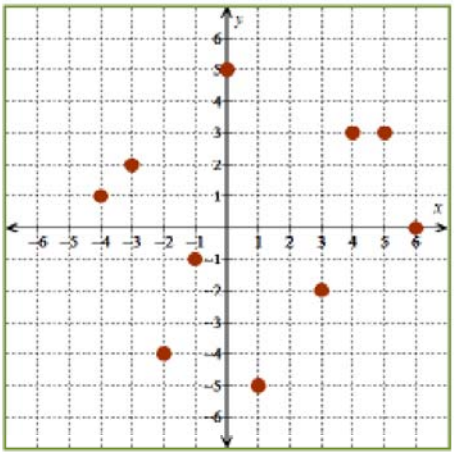

Figure 5: f

Example 8

What are the domain and range of the function shown in Figure 6? Write each set using interval notation.

not including

The domain is $(-5, 1]$.

The range is $[-4, 4]$.

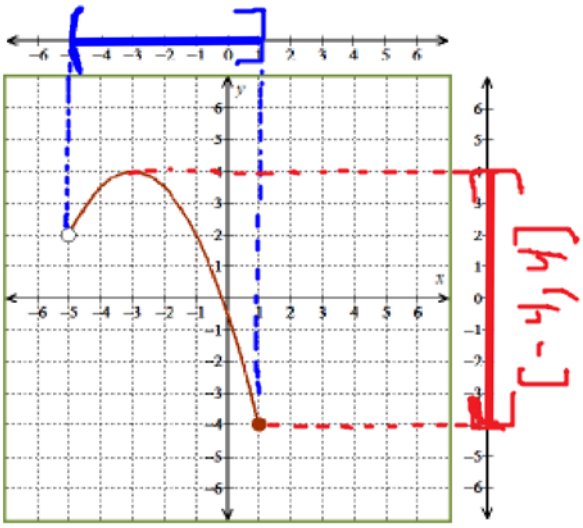

Figure 6: k

Example 9

What are the domain and range of the function shown in Figure 7? Write each set using interval notation.

The domain is $(-\infty, 6)$

The range is $[-3, \infty)$.

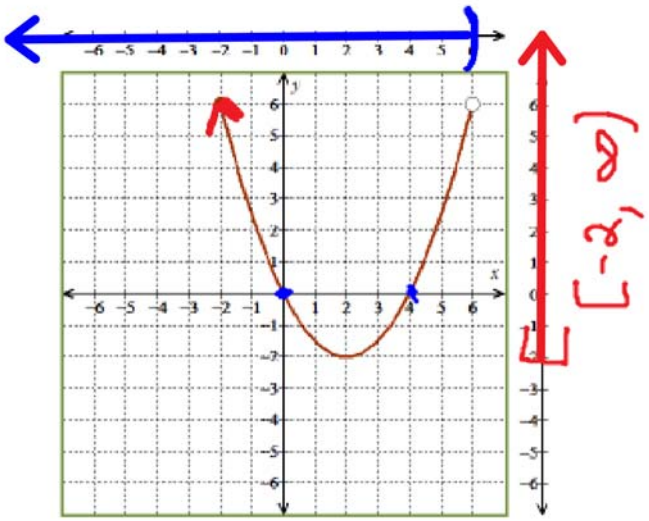

Figure 7: g

Example 10

Answer each of the following questions about the function shown in Figure 7.

a. What is the solution set to the equation $g(x) = 0$?

The solution set is $\{0, 4\}$.

b. What is the smallest function value found on the graph?

(what is the least y-coordinate on the curve)

-2

c. Estimate the function value at 5.

$g(5) \approx 2.5$

d. What is $g(6)$?

$g(6)$ does not exist.